REPORT TO THE CALIFORNIA STATE BOARD OF EDUCATION

Review of High Quality Experimental Mathematics Research

Executive Summary
"At a Glance"

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This document is a summary of our findings reported at greater length in our principal report of January 31, 1998 and our addendum report of March 10, 1998. This report summarizes one hundred ten high quality studies: seventy-seven from our principal report and an additional thirty-three from our addendum report.

There are two sections to this report. The first section is a brief review of our findings, in chart form. We first review categories with multiple support, followed by studies that have a unique focus.

In the second section of this report, we outline the characteristics of an effective mathematics lesson that are strongly supported in our findings.

Section I: Findings

Categories with Multiple Support

Category	**Findings**
Cooperative and/or Peer Work	12 of 13 studies found positive effects. Students varied considerably in terms of grade level and performance levels. The “negative” study produced good results with both cooperative and individual reinforcement. The benefits of peer work derive from (a) cooperative reinforcement systems, and (b) additional scaffolded instruction from peers.
Instructional Design: Didactic versus Discovery Learning	There are three studies in this group. A slight advantage appears to go to *guided* discovery for some content and with higher performing students, and to *didactic* instruction for some content and with lower performing students. No advantages to strictly discovery instruction are indicated in these studies. Several other studies in our review investigated some other aspect of mathematics instruction, but found positive effects for explicit instruction as well.
Instructional Design: Strategy Instruction	11 of 13 studies found positive effects for particular types of strategy instruction. “Strategy instruction” is a very broad category. Some strategy instruction produces positive effects and some does not. The effective strategies identified in these studies incorporated explicit instruction.
Instructional Design: Selection and sequencing of examples	3 of 4 studies found positive effects for selecting and sequencing instructional examples according to principles of concept acquisition. One study found no difference between a programmed and a random sequence for teaching fractions to educable mentally retarded students.

Instructional Design: Mastery Learning	2 studies directly investigated the effects of mastery learning and found positive effects when mastery learning is used in conjunction with other approaches (cooperative learning and elaborated instruction with feedback). One technology study also found positive effects for mastery learning when the modes of initial and remedial instruction (CAI and teacher-directed) were reversed.
Calculators: computation	3 studies found positive effects for using calculators as a supplement to intense programs of computation instruction. One study found hand held calculators useful for higher performing students, and programmed calculators useful for low performing students. (The programmed calculators gave students problems and provided feedback.)
Calculators: problem solving	2 studies showed equivocal results. One did not directly assess the effects of calculator use on students' ability to solve verbal problems, but found other positive effects (e.g., students using calculators performed computations more accurately). The other unequivocally found no negative effects from using calculators with problem solving.
Computers: LOGO programming language	Three studies found positive benefits for using the LOGO programming language in conjunction with learning the concept "variable," motion geometry, and problem solving. One study found the effects of CAI and LOGO to be comparable.
Computers: Computer-assisted Instruction (CAI)	5 of twelve studies found straightforward positive effects for CAI. Two studies found CAI most effective for higher performing students and teacher-directed instruction most effective for lower performing students. One study found CAI combined with cooperative learning to be more effective for lower performing students than CAI alone (which was effective with higher performing students) One study found "plain" CAI more effective than game-oriented CAI, and another found no difference between CAI and conventional instruction, and two studies found negative effects for the use of CAI.
Grouping	We can draw few, if any, straightforward conclusions based upon the studies that focused upon grouping: large versus small groups, heterogeneous versus homogeneous groups. Rather, these studies suggest that grouping interacts with other variables. Any group size and makeup can be effective under some circumstance. One study found positive effects for small class size (15) and a teacher only, compared with a class size of 22 with a teacher and aide. One study found small group cooperative instruction to be more effective with high performing students, and large group individualized instruction to be more effective with low performing students.

Reinforcement and Motivation	6 studies directly investigated the effects external reward systems, and all found positive effects. Four involved lower performing students, and two involved regular classroom middle school students. Two studies directly contradicted one another with respect to the question of whether students or teachers should set learning goals. One study found self-monitoring and conventional adult monitoring of student progress to be equally effective. Another found self-monitoring to be more effective than self-monitoring combined with goal setting.
Assessment	Four studies found positive effects for the use of Curriculum-based Measurement (CBM). One study found better effects using student self-assessment than conventional assessment.
Manipulatives	Four studies investigated the effects of manipulatives upon mathematics achievement. Three of those studies—all conducted in elementary schools—found no benefit. One study, conducted with middle school students studying fractions and ratios, did find positive benefits.

Individually Supported Findings

Focus	**Findings**
Use of literature, child-initiated play and manipulatives.	Effective at Kindergarten with unspecified mathematics content.
Use of "lattice algorithm" for multiplication.	Use of algorithm was more effective than conventional multiplication algorithm.
Use of part-part-whole approach to teaching numbers.	Approach (1+9 or 2+8 or 3+7, etc., = 10) was effective.
Complex fractions versus associative approach to teaching division of fractions.	Complex fractions approach was more effective.
Effects of teacher affect.	Teachers with positive affect bring about better mathematics achievement that teachers with negative affect.
Effects of synthetic and analytic approaches to teaching geometry proofs.	Either a synthetic approach along, or a combination of a synthetic and analytic approach to teaching geometry proofs, produced better results than an analytic approach alone.
Effects of teaching transformational geometry.	Teaching transformational geometry has better effects on learning transformational geometry than not teaching it.
Effects of vocabulary instruction in ratio and proportion instruction.	Use instruction on critical prerequisite vocabulary for solving ratio and proportion problems when teaching ratio and proportion.
Effects of tutoring versus conventional instruction on exponential notation.	Use tutoring instead of conventional instruction to teach exponential notation.
Effects of mental computation on problem solving.	Use mental computation instead of conventional instruction to teach problem solving.

Effects of personalized word problems on problem solving.	Use personalized word problems to teach problem solving.
Effects of direct instruction with and without a small group variation.	Use whole group direct instruction with a small group variation for teaching fractions to students who are internally motivated high performers with a positive attitude toward mathematics, but whole group direct instruction without a small group variation for externally motivated lower performers with a negative attitude toward mathematics.
Effects of progress monitoring.	Use either self-monitoring or external (adult) progress monitoring, as opposed to no progress monitoring.
Effects of incidental versus intentional instruction on number concepts.	Use incidental instruction)a game format) rather than conventional intentional instruction to teach number concepts to students identified as educable mentally retarded.
Effects of three approaches to introducing two-digit numeration.	Use grouping objects in sets of ten and counting by tens, rather than other approaches, to introduce two-digit numeration.
Effects of teachers' written feedback on student homework.	Use specific teacher's comments (feedback) on homework, rather than simply indicating the number correct.
Effects of the Piacceleration[1] program.	Use the Piacceleration program, along with a wide range of exemplars and full mastery criteria, to teach classification, variation, and number conservation at Kindergarten.
Effects of learning and performance goals.	Use learning goals (how to solve problems) rather than performance goals (solving problems) for addition and subtraction of fractions.

Section II: Effective Mathematics Lessons

After having reviewed all one hundred ten high quality studies, we discerned a pattern for effective mathematics lessons from a substantial number of those studies. A large number of studies included a control group that involved "conventional mathematics instruction." Although the designation "conventional" is general, the fundamental attributes of such instruction were fairly consistent across those studies in which it was described. There are basically two "parts" to a conventional mathematics lesson. First, students sit passively and watch as the teacher demonstrates how to do something new.

[1] This is described as having been "developed out of efforts to use learning sets to teach classification, seriation, and number conservation to blind and mild mentally retarded students."

The new material can be a concept, an algorithm, a strategy, a more general heuristic, etc. There is very little indication among these studies of the *quality* of those initial teacher demonstrations. For example, a teacher could simply demonstrate a rote algorithm, or present a new problem type in a way that fosters understanding.

The other consistent part of conventional mathematics lessons is that after passively observing the teacher for a period of time, students then worked problems on their own. Certainly, there must be variation among teachers regarding the extent to which they monitor students during the independent phase of a conventional lesson, give additional instruction, and so on.

Nonetheless, the studies we have reviewed presented lessons that consistently show better achievement results than the conventional model: students passively observing teacher demonstrations of varying quality, then working independently. Another way of characterizing the conventional lesson model is an abrupt shift from nearly 100% “other-regulation” to nearly 100% self-regulation. Specifically, no student cognition is *required* while passively observing a teacher, then abruptly, students are expected to know and independently apply the information newly introduced moments earlier.

Thus, the conventional lesson model looks like this:

Phase 1	**Phase 2**
Teacher demonstrates (Often, teacher works one to four problems.)	**Teacher might or might not monitor, give feedback, etc.**
Students observe passively	**Students work independently**

In contrast, the lesson models for *effective interventions* most frequently followed a three-phase pattern[2]. In the first phase, teachers not only demonstrated, but explained, and asked many questions, checked for understanding, or conducted discussions. In sharp contrast to the conventional model, students were almost always quite actively involved in the instruction during the initial phase. Further, some studies focused upon the quality of initial instruction.

The second phase of high achievement lessons can be characterized as an intermediate stage, between learning something new and applying that new knowledge independently. We could simply refer to this second phase as the "help" phase of an effective lesson, the stage during which students *gradually* transition from teacher-regulation to self-regulation for students. The details of this second phase vary considerably, from students helping one another collaboratively to high levels of teacher help with feedback and frequent "correctives" (additional explanation when students falter). In many cases, this second phase of instruction took up the majority of lesson time. Therefore, not only do these studies distinguish themselves from conventional lessons by utilizing an intermediate "help" phase of instruction, but they do so dramatically, given that such a help phase is a substantial part of effective lessons.

Because, presumably, student independence (self-regulation) is a universal goal of educators, effective lessons usually included a third phase of individual accountability. In

[2] The descriptions of a conventional model uniformly referred to a single instructional lesson. Some of the studies showing positive effects also made reference to a single instructional lesson, but in some cases, the "model of an effective lesson" described herein refers to a series of lessons related to a given topic. For example, some effective problem-solving "lessons" are in reality a series on lessons in which students gradually move through "phases" of an effective lesson over a period of several days.

contrast to the conventional lesson model, in which the majority of lesson time was often taken up with students working independently on worksheets, independent work made up just a small percentage of lesson time in the effective lessons. Often, the only independent activity was some form of an assessment activity. Frequently, individually accountability included an assessment of the students' ability to *generalize* or *transfer* their knew knowledge to untaught applications. With very few exceptions, the studies did not simply assess students' ability to recall knowledge.

In short, effective lessons don't require students to apply new knowledge independently until they have demonstrated an ability to do so successfully.

The general model for effective lessons looks like this:

Phase 1	**Phase 2**	**Phase 3**
Teachers demonstrate, explain, question, and/or conduct discussions	**Teachers, individual peers, and/or groups of peers provide students with substantial help that is gradually reduced**	**Teachers assess students ability to apply knowledge to taught and/or untaught problems**
Students are actively involved, through answering questions and/or discussion	**Students receive feedback on their performance, correctives, additional explanations, and other forms of assistance**	**Students demonstrate their ability to independently recall and/or generalize and transfer their knowledge**