

**REPORT TO THE CALIFORNIA STATE BOARD OF EDUCATION
and**

Addendum to Principal Report

Review of High Quality Experimental Mathematics Research

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Introduction

We have conducted a thorough review of high quality experimental research in mathematics under a contract with the California State Board of Education. The goals and general parameters of this investigation were established in a Request for Proposals issued by the Contractor, the California State Board of Education.

The principal goal was to locate high quality research in mathematics education, to review such research, and to synthesize the findings. The Contractor quite specifically requested that we locate experimental studies in mathematics, and further, that we evaluate studies according to well-established principles of experimental research design. The Contractor's stated use of such a review was that of informing decisions about Mathematics Frameworks, mathematics standards, and mathematics textbook adoptions. The Contractor was particularly interested in studies that could inform (a) instructional practices and (b) scope and sequence of instruction.

Although this report provides substantial empirical evidence relating to effective instruction, none of the high quality experimental studies that we identified focused upon the scope of instruction, nor the sequence of instructional topics. We believe that a review of high quality descriptive correlation studies would be more profitable for informing decisions about scope and sequence.

This report is the product of our six-month investigation. From a total of 8,727 published studies of mathematics in elementary and

secondary schools, we identified and procured 956 articles that satisfied the minimum identification criterion of being an experimental study of mathematics. Of those, only 110 passed the multi-level evaluation criteria we developed to identify high quality studies—high quality in terms of research design.

At the onset of the project, neither we nor the Contractor could accurately predict the eventual scope of the project. In many respects, we were venturing into the unknown. As is common in such circumstances, the scope of the project proved to be greater than either we or the Contractor predicted. The critical process of reviewing the quality of 956 studies and checking the reliability of those reviews was exceptionally labor-intensive. As a consequence, it became necessary to prepare the original report including only 77 of the high quality studies. An addendum was then prepared which contained the findings from the remaining 33 studies.

There are three major sections to this report: Methodology, Findings, and Addendum. The methodology includes two subsections. The first subsection describes our search methodology: How we located 956 articles for critical quality review. The second subsection describes the research design criteria we used to reduce our list to 110 high quality studies.

The second major section of this report describes our findings. The findings are reported in three ways. First, we discuss the findings in categories that we hope will make them meaningful to readers. Next, we "list" the findings from each of the high quality studies, indicating the focus of each study, and minimum critical aspects of the findings of each. Third, we list the variables identified from the studies. We identified 128

instructional variables. Again, for the convenience of the reader, we first list all 128 variables, then reorganize them into categories of similar variables. In reality, many of the variables identified are very similar to one another, differing principally, if not exclusively, by the labels researchers chose to give them. However, by listing the 128 variables, we invite readers to develop categories using alternative criteria.

The addendum contains the findings of the 33 studies remaining after the original report was completed. The results are reported similar to the previous section, with a discussion of the studies first, then a list of the focus and findings for each study.

Finally, there are three appendices to the report. Appendix A is a list of references for the 968 experimental studies we identified for review. The references are first ordered alphabetically, and then in an order based upon how far each study advanced through the research design evaluation process.

Appendix B comprises a complete set of copies of the filled-out coding sheets for all the experimental studies.

Appendix C contains the coding sheets we originally proposed to the Contractor, the final version of the coding sheets we used, and a list of the changes made between the development of the original coding sheets and the final set.

Methodology

Search Methodology

As a beginning point for our research, we used listings of mathematics research published annually in the Journal for Research in

Mathematics Education (JRME). From 1971 (for 1970) until 1995 (for 1994), JRME published listings with citations and brief abstracts. Beginning in 1996, for research published in 1995, the publication of the JRME-financed search of research was taken over by ERIC. The 1997 listing (for 1996 research) was not yet completed and published at the time of this review. The JRME listings were the most logical starting point for the following reasons:

- 1. JRME is a publication of the National Council of Teachers of Mathematics. This professional organization represents by far the largest number of mathematics educators in the country.**
- 2. The JRME listings were “pre-screened,” separating research (in its many forms) from a far more vast body of “non-research” mathematics literature.**

Our goal with the JRME listings was to further reduce the listings to a “sub-listing” of experimental and quasi-experimental research (hereinafter referred to simply as “experimental research”). We screened a total of 4,578 JRME listings published annually between 1971 and 1995, including the ERIC-published 1995 listing. Our screening criteria for the 4,578 JRME listings was broad: If either the title or brief abstract referred to an experimental study and involved elementary or secondary mathematics education, that article “passed” the screening phase, and we attempted to procure the actual article. Research summaries and studies with a single-subject or multiple-baseline design were identified and kept in a separate category, to be reviewed separately if time permitted. In addition, 26 articles were identified as part of a special listing in JRME. This listing of articles

was placed in a separate category. From the 4,578 JRME citations, we identified 736 potential experimental studies. For reliability purposes, two different researchers searched each year. Reliability averaged 96% per year.

A crucial goal of our research review effort was to err on the side of thoroughness. To that end, we also conducted searches of The Educational Resources Information Center (ERIC) database, as well as psychological abstracts (PsycLIT). Each search was conducted separately, and in the following order: first the JRME search, then ERIC, and then PsycLIT. The purpose of that sequence was to avoid the possibility of procuring and evaluating the same article twice.

Prior to beginning the research review, we experimented with different combinations of ERIC search criteria, toward the goal of producing a search that included all (or nearly all) experimental studies. Initially, our search criteria proved to be too narrow. For instance, a search using “mathematics research” as a descriptor overlooked numerous experimental studies in mathematics that we were able to identify. We “reverse engineered” our search by looking up well-designed experimental studies, to see what descriptors had been assigned to such studies. Through a long process of trial and error, we settled on mathematics achievement or mathematics instruction or mathematics research as major keywords, and elementary education or secondary education as keywords. A search of 1974 through the beginning of 1997 produced 3,166 citations that we screened for potential descriptions of experimental studies. We also conducted a simple but extremely broad search using the descriptor mathematics but limited to

elementary and secondary education, which resulted in 12,500 citations from 1974 through March of 1997. A random spot-check of those citations failed to reveal any evidence of experimental studies beyond those identified among the 3,166 citations of our previous search.

From the ERIC citations, 158 were identified as potential experimental studies. Of those, 48 were already among the 736 identified from the JRME search. The other 105 potential experimental studies identified from the ERIC search were not among the JRME listings. Reliability checks were conducted on 20% of the ERIC citations and agreement was approximately 99%.

Our third search was of the PsycLIT database for the years 1990-1996, using the descriptors mathematics, mathematics education, mathematics concepts, or mathematics achievement. This provided 993 references and abstracts to search. The screening process, identical to that employed with the ERIC search, provided 198 articles as potential experimental studies. Of those, 70 had been identified from the previous two searches, and 128 were new citations. Reliability checks were conducted on 20% of the PsycLIT citations and agreement was approximately 95%.

Search	References /Abstracts Searched	Studies Identified	Redundant Studies ¹	Non-Redundant Studies	Studies Procured
JRME	4578	736 (16%)	-	735	
ERIC	3166	158 (5%)	48	105	
PsycLIT 90-97	993	198 (20%)	70	128	
Totals	8727	N/A	N/A	968	956

¹ Studies identified in a previous search.

Review Methodology

From the three searches of JRME, ERIC, and PsycLIT, a total of 968 articles were identified. These articles were procured from the Knight Library at the University of Oregon, and from Interlibrary Loan through both the Knight Library and the FirstSearch database. Twelve of the articles were not procured in time for review. The remaining 956 studies were reviewed to determine their pertinence and validity.

Fifteen reviewers were involved in the screening and review of the studies. At the University of Oregon, reviewers included five Research Associates (Ph.D. research faculty), a Research Assistant, two first-year doctoral students, and a third-year doctoral student. In addition, we relied on a research group at the University of Texas at Austin that included an Assistant Professor and five first- through third-year doctoral students.

A special evaluation tool was developed explicitly for this research review. The tool was designed to reveal if an instructional approach was effective when evaluated using scientifically valid research methods, and to determine under what conditions and for which students the approach produced positive effects on criterial measures. The evaluation tool employed a levels analysis: If a study passed the criteria for one level, it advanced for analysis to the next level.

Level I: Screening Criteria. The first level screened studies to determine if they satisfied minimal construct, internal, and external validity. A study needed to be pertinent to the analysis to pass the construct validity section. That is, it had to use quantitative measurements of mathematics achievement to report the effects of an instructional

approach. The internal validity section included questions on whether the study was of quasi- or true experimental group design, had sufficient information to compute effect sizes, had equivalence of groups at pretest, and used a sample that was representative and unbiased. External validity questions asked whether the study provided enough information about the approach, and whether the approach was implemented in settings representative of actual instructional conditions.

Level II-A: Methodological Adequacy - Internal Validity. Level II-A covered the internal validity of the study more in-depth to determine if differences between groups were attributable to the experimental treatment. It included questions on the experimental design, the nature of the control group, the measures used, treatment bias and fidelity, and competing or confounding variables.

Level II-B: Methodological Adequacy - External Validity. Level II-B consisted of more in-depth information on the representativeness or generalizability of the findings. It included questions on the size of the sample, the number of teachers or interventionists per condition, the number of schools per condition, and the length of the intervention. Studies that passed Level II-B were accepted and advanced for the final analysis and reviewed at Level III for more information.

Level III-A: Approach Efficacy. The third level covered the effectiveness of the approach by analyzing the measures used, determining if scores were statistically and educationally significant, and discerning if the effects were transferred and maintained. When the information was provided, results were tracked by subgroup.

Level III-B: Approach Equity Efficacy Analysis. Using Level III-A, the reviewer determined for which subgroups of students (by socioeconomic status, performance, language, and disability) the approach was effective, and for which it was not.

Articles Identified	968		
Articles Procured	956		
JRME Research Article Listings	26		
Single-subject or Multiple-baseline	64		
Level I: Screening Criteria	Studies Reviewed	Studies Failed	Studies Passed
I-A: Construct Validity	866	305	561
I-B: Internal Validity	561	268	293
I-C: External Validity	293	62	231
Level II: Methodological Adequacy	Studies Reviewed	Studies Failed	Studies Passed
II-A: Internal Validity	231	74	157
II-B: External Validity	157	47	110
Level III: Approach Efficacy	110		

The coding sheets used and a list of changes made to them are provided in Appendix C.

Databases

After studies were reviewed, information on how far they advanced in the review was entered in a database. Accepted studies were sent to a final reviewer who determined the focus and findings of each study, and grouped studies by topic. This information was entered in another database, which was later merged with the first database by an independent contractor.

Findings

These findings are based upon the first 77 high quality research studies that were reviewed, as fully described and defined in the Methods section of this report. The findings for the remaining studies are documented in the Addendum.

We have attempted to be sensitive to the fact that many readers of this report are not mathematics educators. Therefore, we have added footnotes in this section whenever it seemed to us that further specification would be helpful to our readership. That practice has the added advantage of clarifying findings that might well be ambiguous even to mathematics educators. For instance, the term “strategy” is general enough to require greater description in order to interpret the findings on strategy instruction.

We have grouped the findings into the following categories:

Studies on Manipulatives: 4 studies

Studies Related to Work Among Peers: 11 studies

Studies Related to the Design of Instruction: 24 studies

Discovery versus Didactic Instruction: 3

Heuristics and Strategies: 10

Sequencing: 2

Mastery Learning: 2

Other: 7

Studies Related to Computers and Technology: 18 studies

Calculators: 5

Logo: 3

Computers: 10

Studies Related to Grouping: 5 studies

Studies Related Directly to Reinforcement or Motivation Systems: 4 studies

Idiosyncratic Studies: 11 studies

In some cases, a study falls squarely and unambiguously into one category or another. In other cases, it would appear that a study could just have easily gone into a category other than the one to which we assigned it. In those latter cases, we attempted to assign a study to a category based upon the central interest of the researchers who conducted the study. However, the assignment of a study to a category is not crucial; we report herein all the findings of a study, regardless of category.

Finally, we should note that we state findings *in terms of mathematics achievement*. That is, a given study might have shown no benefits of a particular condition on mathematics achievement, while showing some benefits in other areas, such as improved attitude towards mathematics or personal preference for one mode of instruction over another. While factors such as attitude toward mathematics are of crucial importance, the focus of our work has been upon mathematics achievement only. Therefore, when a finding reports “no benefit,” that conclusion refers only to mathematics achievement.

Summary of Findings

Manipulatives

Four studies evaluated the use of manipulatives. One showed no achievement benefit in using manipulatives to teach oddity, seriation, and counting to kindergarten children. Pasnak et al. (1996) showed no

advantage to using manipulatives prior to computation instruction for multiplication computation with low socioeconomic third grade students. Bright et al. (1981) found that a fractions game played either with or without manipulatives and pictorial representations enhanced achievement for fifth and seventh grade students. Moody et al. (1971) found no advantage to using manipulatives to teach multiplication computation at grade three with low-middle socioeconomic students.

The fourth study (Harrison, et al., 1989) showed strong positive benefits for using manipulatives in conjunction with fractions and ratios instruction at grade seven. The control group used “conventional”² mathematics instruction. Such instruction is demonstrated to be ineffective among our high quality studies. In addition, the studies conducted with elementary students found no benefit. Only the study with middle school students showed a benefit.

Clearly, the jury is still out on the use of manipulatives. Far more studies are needed, but moreover, studies are needed that control well for the many variations possible in the use of manipulatives. For instance, the question of *when* to use manipulatives (in relation to computation) should be studied carefully, as well as the interaction among manipulatives and other instructional variables.

² Nearly all studies use the terms “traditional” or “conventional” to refer to instruction during which a teacher demonstrates a solution procedure or strategy with between one and four examples, and then students work several problems independently, on worksheets. Usually, there is a summative quiz of some type (as in the Slavin & Karweit, 1984, study), but that is not always specified. The term “traditional” will be used in this sense throughout this report. In studies, the terms “conventional” and traditional” are often used interchangeably.

Work Among Peers

The eleven studies related to work among peers represents a substantial portion of the total number of high quality studies. Overall, the findings are quite positive for some form or another of cooperative peer study. On the other hand, there is no support for the notion of simply putting students together to work in groups. That is, each study in support of cooperative group work couches that work in a larger, well-structured instructional system.

In each case, teachers directed initial instruction with examples and explanations of new material. The work among peers constituted a “second phase” of the instruction. We could characterize this second phase as a transition from teacher-directed instruction to independent application, or as a transition from “other-regulated” instruction to self-regulated.

The “why” of cooperative peer work is more intriguing than the fact of its effectiveness, in that the “why” provides some basis for generalizing these findings. Two distinct “whys” are discussed throughout the eleven studies.

First, cooperative team work can be used effectively as a motivational technique. Some studies focus almost exclusively upon this aspect of cooperative work. In a group or team setting, reinforcement systems can be *cooperative*, rather than *competitive* or “*individualistic*.” An illustration of a *competitive* extrinsic reward system is grading “on the curve.” In this case, one student’s success spells another student’s failure.

In contrast, an “*individualistic*” external reward system can be illustrated by grading on a scale. In this case, it is theoretically possible for

every student to get an “A” (or an F), but the reality in highly diverse classrooms is that lower achievers are the most likely to get the lower grades.

A distinct advantage of cooperative group work is that of providing external rewards on the basis of group performance. Within a heterogeneous group, individuals are quizzed individually, but actual grades are based upon an average of team performance. Such a system encourages higher performers to assist their lower performing peers.

Two studies (Fantuzzo, et al., 1995, and Heller & Fantuzzo, 1993) expand the motivational potential of cooperative group work by combining it with parental involvement with the reward system. In both cases, the overt inclusion of parental involvement notably enhanced the advantages already inherent in cooperative group work. The nature of the parental involvement in both studies was limited strictly to participation in external rewards. That is, the parents were not involved in direct tutoring. Rather, the parents were kept informed of student progress (in terms of both achievement and effort), and agreed to provide various incentives at home for achieving goals. The research reported here is clearly supportive of cooperative external reward systems—all theoretical discussion aside.

In addition to the effects of cooperative group work on motivation, the other powerful dimension of cooperative group relates directly to instructional interactions. The descriptions of “conventional mathematics instruction” found throughout the high quality studies in this report can be characterized as teacher explanation, followed by independent workbook activity. Ironically, that approach has no theoretical support that we are

aware of, and is soundly discredited by the studies in this report. In essence, going directly from teacher explanations to independent work is asking students to be “other-regulated” briefly, and then to suddenly become completely self-regulated.

Highly structured, interactive group work seems clearly to mediate the transition from initial teacher-directed instruction to self-regulation. In general, students *support* one another in group work. More specifically, that support takes the form of more (and, sometimes, different) explanations, more opportunities to respond, additional opportunities for feedback, and the potential (realized in these studies) for better focus upon the tasks at hand.

One study (O’Melia & Rosenberg, 1994) investigated the effects of doing homework cooperatively, and showed no advantage for that approach, in which students were simply assigned to do homework in groups. In all the other studies that show notable effects for cooperative work, work was done in conjunction with an overall instructional system that controlled for the nature of the interactions in the group setting. Assigning students to do homework in groups, apparently, is not an effective application of cooperative group work.

One study in this group (Slavin, Madden, & Leavey, 1984) used programmed learning as its instructional system, and compared cooperative and competitive reinforcement systems. There was no difference in the achievement between the two groups, although achievement for both groups was high. This suggests that in some

instructional settings, either cooperative or competitive reinforcement systems may be effective.

A particularly interesting aspect of these studies on cooperative work is that all of them included students who were at risk in some sense, either by virtue of being inner city urban students, and/or low SES students, and/or students identified as having special learning needs. These studies show strong positive effects for such students, but moreover, the studies show positive effects for higher performing students working with their lower performing peers, as well.

Design of Instruction

Discovery versus Didactic Instruction. Three studies directly address the question of whether initial instruction should be given via a discovery mode (generally favored by constructivists), a guided discovery mode, or a direct mode (generally favored by cognitive and behavioral-cognitive psychologists). One relatively sophisticated study (Anastasiow, et al., 1970) investigated the impact of using discovery, guided discovery, or didactic instruction when teaching pre-math concepts and principles to poverty-level kindergarten students. The findings varied across the dimensions of learner achievement level and content taught. In short, guided discovery was found most effective for (a) pre-math *principles* only with all students, and (b) both pre-math *principles*³ and *concepts* with high performers only. Didactic instruction was found most effective with (a) pre-math *concepts* for all students, and (b) both pre-math *principles* and *concepts* with low

³ Principles and concepts were not described and illustrated in the study. However, a concept is usually a unique categorization, and a principle is usually a statement of relationships among concepts.

performing students. Discovery was not found to be effective for any content or any group of students.

At the opposite end of the grade-level spectrum, Lackner (1972) found that didactic instruction was more effective than discovery instruction in teaching the derivatives concept to regular classroom urban high school students. However, there was no difference between using either discovery or didactic instruction for teaching the limit concept to those same students.

The third study in this subgroup of instructional design studies (Olander & Robertson, 1973) found no difference between discovery and didactic instruction for teaching a variety of mathematics content to regular classroom fourth grade students. These studies alone shed very little light on the question of discovery versus didactic instruction. A slight advantage appears to go to *guided* discovery for some content and with higher performing students, and to *didactic* instruction for some content and with lower performing students. No advantages to strictly discovery instruction are indicated in these studies.

Heuristics and Strategies. Ten studies investigated the effectiveness of strategy and/or heuristics instruction.⁴ Two of those studies taught overt strategies based upon number fact families.⁵ One such study (Wilson & Sindelar, 1991) involved students identified with learning disabilities in grades two through five. The other (Adetula, 1996), regular classroom students in grades two. In the former case, positive effects were found in

⁴ Heuristics are usually considered to be more general than strategies, but in most cases, the distinction between a relatively general strategy and a heuristic is vague.

terms of problem solving. In the latter, facts acquisition. The study that investigated effects upon students identified as having learning difficulties also found that an easy-to-hard instructional sequence was more effective than either the strategy instruction alone, or an easy-to-hard sequence alone.

Thornton's (1978) study emphasized counting on, doubles, thinking one more or less than a known fact, using ten, and recognition of the commutative property. Over all, regular classroom students in grades two and four learned facts better, and transferred and retained their knowledge better than students in control groups taught conventionally, through an emphasis upon drill and practice.

Another study in this group (Swing & Peterson, 1988) showed the benefits of using elaborative and integrative thought processes for teaching measurement to regular classroom grade five students. The elaborative questions in this strategy required students to observe many facets of each individual problem: concepts, pictures, textbook definitions, manipulatives, etc. The integrative aspects of the strategy required students to engage in concept analysis—the isolation of concept features—and comparison, wherein the features of different concepts were compared with one another. Control students used math fact sheets or conventional instruction.

Two studies summarized here used explicit metacognitive⁶ strategy instruction, and feedback based upon that instruction. One (Cardelle-

⁵ The numbers 2, 5, and 7, for instance, comprise a single fact “family” from which four facts can be derived: $2+5$, $5+2$, $7-5$, and $7-2$.

⁶ Metacognition refers to “thinking about thinking.” Students, for example, might be taught a particular strategy for solving math problems. A metacognitive strategy specifically teaches students to think about the mathematics strategy they were taught, as they are applying it. Rather than asking students questions

Elawar, 1990) effectively taught problem solving to English-as-a-Second-Language students at grade six. Instruction took place in the students' native language. The other study (Cardelle-Elawar, 1995) involved lower performing Hispanic students.

Two studies taught self-instructional strategies⁷. Both studies used similar strategies directed toward learning in general, rather than mathematics specifically. The strategy was based principally upon a sequence of instructional events in which students began with a heavy dependence upon teacher-directed instruction, which was gradually diminished to the point that students worked independently.

One of the two studies (Leon & Pepe, 1983) taught students aged nine through twelve who have been identified with learning disabilities or with educable mental retardation. The other (Genshart & Hirt, 1980) taught regular classroom, grade seven females. Both addressed unspecified mathematics content. Although the focus of both studies was upon the self-instructional strategy, both showed positive effects in mathematics achievement.

Another study (Pereira-Mendoza, 1980) investigated the effects of teaching heuristics as a means of improving mathematics achievement. That study involved grade ten, regular classroom males. The heuristics were described only as “explanation of cases” and “analogy.” The use of

about mathematics, a teacher using a metacognitive strategy would ask questions about the when and how they should apply the problem-solving strategy.

⁷ Self-instructional strategies can be applied to any content area. With respect to mathematics, and in contrast with the “conventional” procedure of jumping suddenly from teacher modeling of a few examples of a new problem type to fully independent seatwork, self-instructional strategies teach students to gradually decrease their dependence on the teacher and other help (e.g., peers), until eventually, students work problems successfully and independently.

those heuristics was effective for teaching algebra and geometry problem solving.

Although eight studies provide strong support for overt strategy instruction, one study in this group (Christensen, 1991) found that fact practice, combined with fluency building, produced better effects than strategy instruction. It is possible that this apparent contradiction is the result of the specific strategies employed in the study: counting, near doubles, and near ten strategies. Because two studies based upon number families had positive effects, the number families strategy might be more effective. While overt strategy instruction is generally supported in this group of studies, further attention should be given to the efficacy of particular strategies. In addition, the combination of practice and fluency building with a research-based strategy should be investigated by researchers as a potentially powerful combination.

Sequencing. Two studies investigated the effects of instructional sequencing. One study (Petty & Jansson, 1987) found positive effects for logical sequencing with regular grade six students learning the concept of parallelogram. In a logical sequence, examples are carefully arranged to show students the range of examples covered by a concept, and to show minimal discriminations with examples falling outside the range of the concept. In the other study (Brown, 1970), a similar approach to sequencing proved effective with regular grade ten students studying trigonometry.

Mastery Learning. Two studies investigated mastery learning, along with other instructional factors. The principal identifying feature of mastery learning is that students demonstrate mastery of material being

learned before moving on to study new material. One study (Tenenbaum, 1986) investigated the impact on both higher and lower mental processes of what the researcher termed “quality instruction.” The conditions investigated were conventional instruction, mastery learning with corrective feedback, and a more elaborate instructional condition. In addition to mastery learning and corrective feedback, the more elaborate condition included "enhanced cues" and participation. Enhanced cues are detailed instructions to students on what they are to learn and what they are to do. Participation refers to active involvement of the student during new learning—as opposed, for example, to passively watching teacher modeling. This study found the most positive effects for the more elaborate condition, followed by the mastery learning plus corrective feedback condition.

The other study involving mastery learning (Mevarech, 1991) also studied the effects of cooperative learning, as well as mastery learning and cooperative learning combined. Both mastery learning alone and cooperative learning alone resulted in more positive effects than conventional instruction, but the best effects resulted from a combination of mastery learning and cooperative learning, adding to the support for cooperative learning discussed above. The study was conducted with low socioeconomic regular classroom students at grade three.

Other Instructional Design Studies. Each of the remaining studies in this summary of instructional design studies is unique.

Hiebert and Wearne (1993) investigated the effects of instructional tasks and student discourse on the learning of place value, and addition

and subtraction of whole numbers, for both suburban and rural grade two students. Positive effects were found for (a) having students work on fewer tasks (than in conventional instruction), but in greater depth, and (b) engaging in collaborative student-centered discussion among students, including longer responses to teachers' questions. The second of the two findings was consistent with the cooperative learning findings and the elaborated mastery learning study, in that it supported the notion of active student participation in their instruction. When this study is combined with those others, it appears that active student participation can be achieved through the frequent questioning of groups or individuals, through peer interactions, or through student-centered discussions. The child-centered discussion study and the elaborated mastery learning study were conducted with regular classroom students. Most of the cooperative learning or peer-related studies included lower performing and regular classroom students alike.

Sindelar et al. (1984) investigated the effects of the amount of teacher-directed time allocated to half-hour lessons with regular classroom students learning exponents. The interesting finding is that either 25% or 75% teacher-directed instructional time (with the remaining time going to independent work) proved more beneficial than 50% teacher-directed time. The authors of the study concluded that students need *sustained* time in either teacher-directed instruction or independent work.

Another study (Beady & Slavin, 1981) investigated the effects of "focused instruction" upon learning unspecified mathematics content among inner city students at grade seven. Focused instruction is a more

highly structured variation of the Missouri Mathematics Project developed by Good and Grouws. The focused instruction showed positive effects, whether accompanied by individual incentives or competitive incentives.

One area of instructional design is that referred to as “advanced organizers.” This refers to a set of techniques that can be used before instruction on new material begins. One such technique is that of administering a pretest prior to new instruction, then giving students feedback on their performance—both before instruction begins in earnest. The one study in this group that investigated the effects of pretest feedback (Lashier & Wren, 1977) showed no benefits for the approach.

Fuchs et al. (1990) combined skills analysis with curriculum-based measurement for teaching basic math facts to students in grades three through nine who were identified with learning disabilities. Curriculum-based measurement includes—among many other features—frequent assessment of student progress. The skills analysis component allowed teachers to specifically identify strengths and weakness of each students, based upon their performance during the frequent assessment. The combination of curriculum-based measurement with skills analysis showed positive effects.

A unique study in this group (Rudnitsky et al., 1995) investigated the use of writing, combined with teacher-directed instruction on problem structures and cooperative learning groups. The study was conducted with regular classroom students in grades three and four who were studying verbal problem solving. It is the writing component that sets this study apart from others. That component involved having students

create—write—problems from specific categories of problem types, such as combine problems and compare problems. The study showed positive benefits for this writing approach to problem-solving, and adds strength to the studies cited above on cooperative learning and explicit teacher-directed strategy instruction.

In another study (Schunk & Hanson, 1985), the effectiveness of teacher models versus successful student models was investigated for teaching unspecified mathematics content to low socioeconomic grade three students. The study eliminated a great deal of potentially confounding variability by using a video tape of a teacher modeling new instructional material, and video tapes of a student demonstrating the same model.

The successful peer model proved more beneficial than a teacher model (which, in turn, proved more beneficial than no model at all). The students in the study were low performing regular classroom students in grades two through four. The mathematics content was subtraction with regrouping. The authors of the study conclude that peer models are more successful for low performing students because such students develop more confidence in their own ability when they see that “one of their own” can master the new content.

Computers and Technology

Calculators. Five studies investigated the use of calculators in conjunction with mathematics instruction. Two of those investigated the impact of using calculators in conjunction with computation instruction, and both found positive effects. The students in one study (Schnur & Lang, 1976) were from low socioeconomic backgrounds, ESL students, low

performing students, and regular classroom students, ages nine through fourteen. The other study was with regular classroom students at grade three. The latter study (Standifer & Maples, 1981) compared the use of “programmed” calculators with regular calculators, and found better effects for using regular calculators instead of programmed calculators, and programmed calculators were found to be better than no calculators at all. (The programmed calculators were instructional tools that required students to work particular problems and solve them.)

Two studies investigated the effects of using calculators in conjunction with problem-solving instruction. One (Wheatley, 1980) showed positive effects with regular classroom students at grade six. The other (Szetela, 1987), found benefits to overtly teaching problem-solving strategies, with or without calculators.

The last study in this group (Gaslin, 1975) investigated the use of calculators to convert fractions to decimals before performing fractions computations. Positive effects were found. The usefulness of this study, however, is dependent upon one’s willingness to dispense with teaching fractions computation without first doing a conversion to decimals.

The results on the use of calculators for teaching computation appear to be the most reliable, with qualifications. In both studies, calculators were used as supplements to concerted efforts to teach computation without using calculators. Schnur and Lang had students in the calculator group work one third of their problems using calculators. Students did the other two-thirds of their problems without using calculators, but then checked their results with calculators. Standifer and Maples had students use

calculators for about eight to ten minutes out of a fifty-minute period, primarily for checking answers and doing drills.

Because of what was actually tested and how results were reported, the results of the two studies on problem solving with calculators are equivocal. Wheatley assessed the number of problem-solving process students used, computational accuracy, and whether students used good strategies that would lead to a correct solution, irrespective of computational accuracy. The students using calculators used more problem-solving processes, and performed much better on computation—but the calculator group used calculators during the computation test, while the non-calculator group did not. The non-calculator group did somewhat better on the *production* test—the assessment of whether students used good strategies that would lead to a correct solution, regardless of computational accuracy. A test that is conspicuous because of its absence was one that reports simply on how many students in each group solved the test problems correctly.

Szetela tested computation and three types of problems, in order of complexity: simple translation problems, process problems, and complex problems. Students were tested mid-year and at the end of the year. The most robust findings were that the calculator groups did not experience any degradation in their computational ability without using calculators, and a strong advantage in terms of *attitude* (but not achievement) for the calculator group.

Logo. Three studies investigated the impact of using the Logo programming language as a means of teaching various aspects of

mathematics, and all three found positive effects. In one (Ortiz & MacGregor, 1991), grade six regular classroom students learned the concept “variable” more effectively than through conventional textbook instruction on that concept. In another (Johnson-Gentile et al., 1994), Logo was used to teach motion geometry to regular classroom students in grades five and six, and to do so more effectively than a conventional approach, or more effectively than with a focus upon motion geometry but without using Logo. The third study in this group (Nastasi et al., 1990) found positive benefits to using Logo, in conjunction with teacher-directed instruction and students working in pairs, as opposed to working solely on a problem-solving CAI program.

Computer-assisted Instruction (CAI). The studies investigating the effects of CAI on mathematics achievement are generally positive, with some qualifications. Two studies differentiated the effects of CAI on average versus lower performing students. One (McCollister et al., 1986) compared using CAI or teacher-directed instruction to teach numbers and counting to kindergarten children. CAI had more positive effects with students already familiar with numbers and counting, while teacher-directed instruction proved better for those students who were just beginning to learn numbers and counting. The other study (Mayes, 1992) investigated the effects of using CAI heuristics and strategy instruction on teaching problem solving to students in grades nine through twelve. That study found the guided discovery CAI condition better for average and high performers, but found explicit teacher-directed heuristics and strategy instruction more effective for lower performers.

Five additional studies found CAI treatments to be effective. One (Fletcher et al., 1990) involved rural grade three and five students learning computation via CAI drill and practice. That study also had students working in pairs at the computer, and another important finding was that working in pairs at the computer resulted in cost effectiveness relative to gain. Another study (Mevarech et al., 1991) found that having students work in pairs on a CAI program used to teach unspecified content to low socioeconomic students was superior to having students work individually on the same CAI program. Both studies add additional support to the findings on peer collaboration reported earlier.

Another study (Hatfield & Kieren, 1972) found that teaching computer programming with the computer positively influenced mathematics achievement over conventional instruction alone with students at grades seven and eleven at a university high school.

Shiah et al. (1995) specifically targeted students in grades one through six who had been identified with learning disabilities. Variations of CAI were used to teach unspecified content. The finding is that CAI improved achievement, with or without explicit strategy instruction, and with or without animations, which are generally presumed to improve motivation.

One study (Ball, 1988) found CAI effective for teaching fractions to regular classroom students in grade four, when used in conjunction with fractions strips, and in comparison to conventional instruction.

One study in this group (Ferrell, 1986) did not favor CAI. Regular classroom students in grade six were taught unspecified content using

either immersion in CAI or conventional instruction. No differences were found between treatments. Finally, the last study in the group (Okolo, 1992) found that either CAI games or conventional drill and practice were equally effective for teaching basic arithmetic facts to students in grades four through six who had been identified with learning disabilities.

Grouping

One study in this group (Mason & Good, 1993) found positive effects for whole group instruction, when used in conjunction with the “active learning” system advocated in the Missouri Mathematics Project. The whole group instruction was contrasted with dividing students into two smaller groups. Mathematics content was unspecified, and students were in grades four through six regular classrooms. A qualification to this study, however, is that the whole groups themselves were the product of grouping. That is, students were placed in the whole groups based upon entry achievement level. The finding, then, suggests that a further refinement of grouping into two smaller groups is not as effective as whole group instruction with relatively homogeneous large groups.

Hooper and Hannifin (1991) directly investigated the question of homogeneous and heterogeneous⁸ grouping while teaching a contrived mathematics content⁹ to students in grades six and seven. Heterogeneous grouping proved more effective for lower performers, while homogeneous grouping proved more effective for high performers. These findings present a possible dilemma to practitioners. Presumably, lower performers benefit

⁸ A heterogeneous group is made up of students of varying ability and/or achievement levels. In a homogeneous groups, students are at relatively similar ability and/or achievement levels.

from heterogeneous grouping because of the additional support they receive from the high performers. Yet, the study suggests that the high performers themselves achieve best in homogeneous groups.

Achievement levels also appear to influence large versus small group instruction, based upon a study (Peterson et al., 1981) with geometry students in fourth and fifth grades. That study showed no difference between large and small group instruction when the class comprises a broad range of student performance. However, small group instruction proved superior for the both the highest students and the lowest. Another study (Swing & Peterson, 1982) with grade five students learning fractions and division shows similar findings, with the additional qualification that students are trained specifically on how to work in small groups.

Finally, one study (Parr et al., 1981) found no difference between whole group and individualized instruction. The subjects were grade two bilingual students learning unspecified mathematics content.

Reinforcement or Motivation Systems

Four studies directly investigated reinforcement or motivation systems. Walker and Hops (1976) found positive effects for reinforcing either academic performance, or behavior facilitative of academic performance, or both. That study was conducted with average and low performers in grades one through three, with unspecified mathematics content. Another study (Glavin et al., 1971) found positive effects for a highly structured reinforcement system when teaching unspecified mathematics content to students who had been identified as having emotional or behavioral

⁹ The researchers used a novel arithmetic symbol system to reduce the possibility of prior knowledge

disorders. The reward system in that study was implemented in a special pull-out resource room.

The other two studies in this group investigated goal formulation, with an emphasis upon the relative effectiveness of teacher-selected goals versus student-selected goals. One (Fuchs et al., 1989) found in favor of student-selected goals. That study involved students who had been identified as having learning disabilities in grades nine through twelve, with mathematics computation. The other study (Hannafin, 1981), however, found no difference between self-selected and teacher-selected goals for regular classroom students studying unspecified mathematics content in grade eight.

Idiosyncratic Studies

The studies in this group are not necessarily idiosyncratic in the sense of being “odd,” but in the sense that they investigate some aspect of mathematics instruction that is not investigated in any other study in our corpus of high quality experimental studies. In most cases, these studies add support to other findings in other categories.

Curriculum-based Measurement¹⁰ (CBM). Fuchs et al. (1991) found positive effects from using CBM along with an expert computer-based system that teachers used to ensure that remediation or reteaching was based upon best practices. The study investigated students identified as having mild or moderate learning disabilities and students identified as

confounding their findings on grouping.

¹⁰ Curriculum-based Measurement (CBM) is a system entailing frequent assessment of student progress, and frequent adjustments of instruction based upon the assessment. An expert computer-based system provides specific, detailed, “expert” instructional strategies for teachers to use to improve student performance.

having learning disabilities or emotional disturbances. The students ranged from grades two through eight, and studied unspecified mathematics content.

This study lends support to this particular application of technology, and is supported in turn by other studies discussed above that involved CBM in particular, and frequent assessment in general.

Children's Literature. This study (Jennings et al., 1992) investigated the use of children's literature, along with child-initiated play and manipulatives, upon the learning of unspecified mathematics concepts among regular classroom and low socioeconomic kindergarten students. The findings were positive for this approach, compared with conventional instruction.

Lattice Algorithm. The finding of this study (Hughes & Burns, 1975) was that the use of the "lattice algorithm," an unconventional but very specific method of doing multi-digit multiplication. Practitioners interested in alternative algorithms might find this study of interest. A feature of the lattice algorithm is that it breaks the multiplication process into more steps than the conventional algorithm, and requires the application of easier mathematics.

The finding was positive for the lattice algorithm for students at grade four in regular classrooms.

Part-Part-Whole. This study (Fischer, 1990), conducted with middle class kindergarten students learning numerals, found positive effects for using a part-part-whole approach. That approach made explicit to students the various combinations of number sets that add up to a given number: $7 =$

1+6 or 2+5 or 3+4 or 4+3 or 5+2 or 6+1. This study lends some additional support to the idea of teaching strategies based upon number families, discussed above.

Homework. Gray and Allison (1971) found no benefit to assigning homework to grade six middle class students studying fractions computation.

Matching Amount of Instruction to Learner Preferences for Amount of Instruction. When teaching geometry to average and lower performing middle class students in grades nine and ten, Hannifin and Sullivan (1996) found no benefit to attempting to match student preferences for amount of instruction to the amount of instruction actually given.

Teaching Complex Fractions. This study (Ingersoll, 1971) compared using the associative property of mathematics with a complex fractions approach, for teaching division of fractions to low and average learners at grade six. The study found in favor of using a complex fractions approach, which appears to better demonstrate the relationships involved in fractions division than an approach based upon the associative approach. A critical prerequisite to using the complex fractions approach is the identify principle of mathematics. Students must learn that they can multiply any number by one without changing the number, and must learn the many manifestations of “one.”

Teacher Affect. This study (Moore, 1988) provides empirical evidence for something that every mathematics educator, every educator, and adults of all descriptions probably already believe: teachers with positive affect bring about better mathematics achievement than teachers with negative

affect. The students in this investigation were in grades seven and eight. They were low performers and students identified as having learning disabilities, and they studied unspecified mathematics content.

Use of Mastermind to Teach Logic. Bright et al. (1983) found no benefit to using the game Mastermind to teach formal logic at grades six and eight with regular classroom students.

Overt Program to Teach Transformational Geometry. This study (Williford, 1972), conducted at grades two and three with average and high performing students, found that teaching transformational geometry overtly resulted in better achievement on a measure of transformational geometry than if transformational geometry were not taught at all. This study seems to support the more general notion that if we want students to learn something, we should teach it to them.

Proof Construction. Carroll (1997) found that either a synthetic approach to geometric proof construction, or a combination of synthetic and analytic approaches, resulted in better performance than the use of an analytic approach alone. The study was conducted with regular classroom students in grade seven. Because a synthetic approach is the more direct route to teaching geometric proofs, this study lends some support to those instructional design studies that found in favor of explicit instruction (as opposed to the more discovery-oriented analytic approach).

Listing of Findings

The following types of information for each study are reported in the listing:

Focus. This a brief statement of the major focus of the study, stated in terminology used within the study, often in the title.

Finding. We report several key aspects of each study, in a list format:

A statement of the central recommendation(s) to practitioners, based upon the study. We state this in terms of an action, whenever possible. For instance, most of these statements begin with the word “use,” then identify the principal variable or variables supported by the study. In some cases, in which the study shows no effect for targeted variables, statements simply say so.

When applicable, we identify any major variables that were not the principal target of a study, but that were present, along with targeted variables. Such statements begin with the words, “along with.” In some cases, the “along with” conditions were present across all conditions. In other cases, they were present in only the experimental conditions. We make that distinction in the section of this report on Variables, but not in the summaries of findings, reported below.

When applicable, we have included a statement indicating the condition utilized in control groups. Most frequently, when such a condition is identified, it is along the lines of “conventional instruction.”

Next, we indicate the area of mathematics that students worked on in the study (e.g., algebra, basic facts, subtraction with regrouping, etc.). In some cases, no discrete mathematics content was specified. We indicate the content for such studies as “unspecified.”

Grade level, or in some cases, students' ages are also included

Finally, we give an indication of the types of students involved in the study. This usually refers to whether students were in general education, and if not, the category of special education.

Manipulatives

Focus Effects of using manipulatives to teach oddity, seriation, and number conservation to kindergartners.

Finding There was no benefit in terms of academic performance to using manipulatives rather than conventional instruction for teaching oddity, seriation, and number conservation at kindergarten with lower performing poor urban and blue collar suburban children

Short Citation Pasnak, Hansbarger, Dodson, Hart, 1996

Focus Effects of varying manipulative game restraints on learning to order fractions.

Finding Use a fractions game along with no manipulatives or no pictorial representations

OR

along with manipulatives or pictorial representations rather than no game for ordering fractions at grades five and seven with regular classroom students

Short Citation Bright, Harvey, & Wheeler, 1981

Focus Use of manipulatives to teach fractions and ratios.

Finding Use manipulatives
 along with example-rule instruction
 rather than no manipulatives
 (conventional instruction)
 for ratios and fractions
 at grade seven
 with regular classroom students

Short Citation Harrison, Brindley, & Bye, 1989

Focus Effects of activity-oriented
 instruction (manipulatives) upon
 computation

Finding There is no advantage to using
 manipulatives
 rather than conventional instruction
 for multiplication computation
 at grade three
 with low-middle socioeconomic students

Short Citation Moody, Abell, Bausell, 1971

Studies Related to Work Among Peers

Focus Mastery Learning and Student Teams

Finding Use Student Team Learning¹¹
 along with clear specification of
 objectives and frequent assessment of
 progress
 rather than a) conventional
 instruction, or b) mastery learning¹²

¹¹ Student Team Learning is an approach to rewards in which students, following initial teacher-direct instruction, work in groups of four. Each student in a team is assessed individually, then individual team scores are averaged. Each student on a team is graded based upon the team average.

for general mathematics
 at grade nine
 with low performing inner city
 students

Short Citation Slavin & Karweit, 1984

Focus Effects of peer tutoring on arithmetic
 facts for students with LD

Finding Use peer tutoring
 along with a) feedback and
 corrections, and b) reinforcement
 rather than conventional instruction
 for basic math facts
 at (no grades given: ages 6-10)
 with students who have learning
 disabilities

Short Citation Beirne-Smith, 1991

Focus Effects of cooperative, competitive,
 and individualistic conditions on
 children's problem solving

Finding Use cooperative learning¹³
 rather than competitive or
 individualistic learning
 for verbal problem solving
 at grade one
 with urban students

¹² The most prominent feature of “mastery learning” is reteaching, or correctives, for students who do not reach a given mastery criterion initially—usually, 80%

¹³ In the cooperative condition, students working collaboratively in groups are rewarded based upon group performance. In a competitive condition, students work alone and a student’s grade is dependent upon those of other students (“grading on a curve”). In individualistic conditions, students work alone and are rewarded or graded on a “scale,” independently of other students.

Short Citation Johnson, Skon, & Johnson, 1980

Focus Effects of cooperative learning and individualized instruction on mainstreamed learners with learning difficulties and regular classroom students

Finding Use Team Assisted Individualization combined with individualized instruction

OR

Use individualized instruction alone rather than conventional textbook-based instruction for unspecified content at grades three through five with regular education students and inclusion students with learning handicaps

BUT

There were no achievement differences between the conventional control and the two experimental interventions with the inclusion students with learning handicaps

Short Citation Slavin, Madden, & Leavey, 1984

Focus Parent involvement and peer tutoring effects on mathematics achievement

Finding Use a combination of peer tutoring in small groups and parental participation along with feedback with corrections and mastery before moving on

rather than independent practice in
small groups
for unspecified content
at grades four and five
with at-risk students

Short Citation Fantuzzo, Davis, & Ginsburg, 1995

Focus Effects of structured reciprocal peer
tutoring and group rewards on
mathematics achievement

Finding Use structured reciprocal peer
tutoring and group rewards
along with feedback and corrections,
and mastery before moving on
rather than either approach
separately, or conventional
instruction
for unspecified content
at fourth and fifth grade
with at-risk students

Short Citation Fantuzzo, King, & Heller, 1992

Focus Effects of cooperative homework teams
on mathematics achievement for
secondary students with mild
disabilities

Finding There is no advantage to using
cooperative homework groups
rather than no cooperative homework
groups
for (unknown content)
at grades six through eight
with students with mild learning
difficulties

Short Citation O'Melia & Rosenberg, 1994

Focus Effects of peer tutoring and parental involvement on mathematics achievement

Finding Use peer tutoring¹⁴ and parental involvement

OR

Use peer tutoring alone rather than no peer tutoring or no peer tutoring and parental involvement for varied computation at grades four and five with low SES black children

Short Citation Heller & Fantuzzo, 1993

Focus Effects of cooperative learning

Finding Use cooperative team learning along with conventional instruction rather than conventional instruction alone for plane geometry at grades ten through twelve with regular classroom students of varying achievement

Short Citation Nichols, 1996

Focus Effects of cooperative versus competitive learning methods on mathematics achievement

¹⁴ The peer tutoring focused on group rewards. The extent of the parental involvement was limited to providing external awards for achievement, rather than direct involvement in tutoring.

Finding Use programmed learning¹⁵
 along with either competitive or
 cooperative learning methods
 for teaching unspecified mathematics
 content
 at (no grade given) ages seven through
 eleven
 with low performing students

Short Citation Rzoska & Ward, 1991

Focus Effects of peer-assisted learning
 strategies on the mathematics
 achievement of students with varying
 learning histories

Finding Use peer assisted learning strategies
 (PALS)
 along with teacher-directed initial
 instruction and frequent monitoring,
 feedback, and correctives
 for addition, subtraction,
 multiplication, and division
 computation
 at grades two through four
 with students ranging from low to high

Short Citation Fuchs, Fuchs, Phillips, Hamlett, & Karns,
 1995

Studies Related to the Design of Instruction

Discovery versus Didactic Instruction

Focus A comparison of guided discovery¹⁶,
 discovery, and didactic teaching of
 pre-math concepts to kindergarten

¹⁵ The focus of the study was cooperative versus competitive reinforcement systems. Programmed learning was used in both conditions. Both conditions improved achievement substantially, but there was no difference between the two treatments.

¹⁶ Guided discovery is defined as a great deal of cueing on examples, leading to rules. In didactic instruction, the rules are given before the examples. In discovery, examples only are used, without cueing.

poverty children

Finding

Use guided discovery
rather than discovery or didactic
instruction
for pre-math principles
at kindergarten
with all students

AND

Use guided discovery
rather than discovery or didactic
instruction
for pre-math principles and concepts
at kindergarten
with high performers

AND

Use didactic instruction
rather than discovery or guided
discovery
for pre-math concepts
at kindergarten
with all students

AND

Use didactic instruction
for pre-math principles and concepts
at kindergarten
with low performing children

Short Citation

Anastasiow, Sibley, Leonhardt, & Borich,
1970

Focus

Effects of inductive and deductive
teaching methods on teaching limit and

derivatives in calculus

Finding

Use deductive teaching
along with programmed learning
rather than discovery teaching
for teaching the derivatives concept
in calculus
at high school
with urban students

AND

There was no difference between
deductive and discovery conditions
For teaching the limit concept in
calculus
for ordering fractions
at grades five and seven
with regular classroom students

Short Citation

Lackner, 1972

Focus

Effects of discovery and expository
methods on teaching fourth grade
mathematics

Finding

There is no advantage¹⁷ to using
either discovery or expository
instruction
for unspecified mathematics content
at grade four
with regular classroom students

Short Citation

Olander & Robertson, 1973

¹⁷ No advantage, in terms of mathematics achievement. The discovery students showed a better attitude toward mathematics.

Heuristics and Strategies

Focus Using direct instruction to teach word problems to students with learning disabilities

Finding Use a) explicit strategy instruction based upon number families, and b) an easy-to-hard instructional sequence¹⁸ along with direct presentation techniques¹⁹ rather than explicit strategy instruction alone, or sequencing alone for addition and subtraction problem solving at grades two through five with students identified as having learning disabilities

Short Citation Wilson & Sindelar, 1991

Focus Effects of think strategies on basic fact acquisition

Finding Use thinking strategies rather than conventional instruction for math facts at grades two and four with regular classroom students

Short Citation Thornton, C.A., 1978

Focus Effects of self-instructional training for remediating arithmetic deficits

Finding Use a self-instructional strategy²⁰

¹⁸ Sequence of problem types was: simple action, classification, complex action, and comparison.

¹⁹ Identified techniques included: choral responses, frequent questions, scripted lessons, and reinforcement.

rather than systematic instruction
without the self-instructional
strategy

for unspecified content

at (grades not given) ages 9-12

with students who have learning
disabilities and students who are
educable mentally handicapped

Short Citation Leon & Pepe, 1983

Focus Effects of elaborative²¹ and
integrative thought processes in
mathematics learning

Finding Use elaborative and integrative
thought processes
along with the active teaching model²²
rather than fact sheets, or
conventional instruction
for measurement
at fifth grade
with white middle-class students

Short Citation Swing & Peterson, 1988

Focus Effects of strategy instruction versus
fact practice upon the acquisition of
math facts

Finding Use practice
along with fluency building

²⁰ The elements of a self-instructional strategy include: teacher modeling of strategy, students and teachers applying strategy together, overt student application of strategy with teacher observation and assistance, student application while “whispering” strategy (allowing for teacher assistance), and covert (independent) application of strategy. This can be viewed as a sequence of gradually fading teacher intervention. “Systematic instruction” is not well defined, but includes an easy-to-hard sequence of problems.

²¹ Elaborative questions required students to look at many facets of the same problem: concepts, pictures, textbook definitions, manipulatives, etc. Integrative questions required students to do analysis (isolation of concept features) and comparison (juxtaposition of concept features).

²² Good, T., Grouws, D., & Ebmeier, H. (1983). *Active mathematics teaching*. New York: Longman.

rather than explicit strategy
instruction²³
for teaching basic facts
at grade two
with regular classroom students

Short Citation Christensen, 1991

Focus Effects of teaching counting and
thinking strategies on math fact
acquisition.

Finding Use overtly taught thinking and
counting strategies
along with explicit teacher-directed
instruction and an emphasis upon fact
families
rather than conventional fact practice
for teaching addition and subtraction
math facts
at grade two
with regular classroom students

Short Citation Adetula, 1996

Focus Effects of feedback on verbal problem-
solving for bilingual students

Finding Use metacognitive feedback designed to
get students to focus upon problem-
solving strategies
along with strategy instruction,
external rewards, teacher modeling,
and scaffolding
rather than conventional instruction
for problem solving
at sixth grade

²³ The explicit strategies taught were limited to: counting, near doubles, and near ten.

with English-as-a-second-language students²⁴

Short Citation Cardelle-Elawar, 1990

Focus Effects of metacognitive instruction on the achievement of low performers

Finding Use overt metacognitive strategy instruction²⁵ along with teacher-directed strategy instruction and teacher monitoring rather than no strategy instruction for problem-solving at grades three through eight with low performing (Hispanic) students

Short Citation Cardelle-Elawar, 1995

Focus Effects of self-instructional training on mathematics achievement for females

Finding Use self-instructional training²⁶ along with tutoring rather than conventional tutoring for unspecified mathematics content at grade seven with regular classroom females

Short Citation Genshaft & Hirt, 1980

Focus Effects of teaching heuristics to solve novel problems

²⁴ Instruction was in native language.

²⁵ Steps in strategy: translation, integration, solution planning and monitoring, and solution execution.

²⁶ Steps: cognitive model, overt external guidance, overt self-guidance, covert self-instruction.

Finding Use heuristics²⁷
 along with self-study
 rather than no heuristics
 for algebra and geometry problem
 solving
 at grade ten
 with regular education boys

Short Citation Pereira-Mendoza, 1980

Sequencing

Focus Sequencing of examples and non-
 examples

Finding Use well-sequenced examples and non-
 examples
 rather than conventional random
 examples
 for teaching the concept parallelogram
 at sixth grade
 with regular classroom students

Short Citation Petty & Jansson, 1987

Focus Effects of logical sequencing on
 learning complex problem-solving

Finding Use logical sequencing
 along with programmed learning
 rather than random sequencing
 for teaching trigonometry
 at grades ten and eleven
 with regular classroom students

Short Citation Brown, 1970

²⁷ The heuristics used are identified only as “explanation of cases” and “analogy,” but no specific examples are given. “Heuristics” are general procedures, usually more general than “strategies.”

Mastery Learning

Focus	Effects of quality instruction on higher and lower mental processes
Finding	Use a) enhanced cues ²⁸ , b) participation ²⁹ , c) reinforcement, and d) feedback with correctives to improve achievement on lower and higher mental processes along with mastery learning rather than a) conventional instruction, or b) mastery learning and feedback with correctives alone for algebra at grade nine with middle-income students
Short Citation	Tenenbaum, 1986

Focus	Effects of learning mathematics in different mastery environments
Finding	Use a combination of mastery learning and cooperative learning rather than either alone, or neither
	OR
	Use either cooperative learning or mastery learning rather than neither for unspecified mathematics content at grade three with low SES regular classroom students

²⁸ Instructions provided by teacher or materials on what is to be learned and what the learner is to do.

²⁹ Practice that increases learner involvement

Short Citation Mevarech, 1991

Other Instructional Design Studies

Focus Effects of instructional tasks and classroom discourse on students' learning

Finding Use a) fewer tasks worked in greater depth, and b) collaborative student-centered discussion among students, including longer responses to teachers' questions rather than conventional instruction for place value, and addition and subtraction of whole numbers at second grade with suburban/rural students

Short Citation Hiebert & Wearne, 1993

Focus Effects of amount of teacher-directed time in a lesson on learning exponents

Finding Use 25% or 75%³⁰ of lesson time for teacher-directed instruction along with scripted lessons rather than 50-50 teacher-directed and independent lesson time for teaching exponents at grade four with regular classroom students

Short Citation Sindelar, Gartland, & Wilson, 1984

Focus Effects of focused instruction and student evaluation on mathematics

³⁰ The more general conclusion is that students should work on activities that are sustained for a majority of the lesson period, whether teacher-directed or independent. Students in the 75% teacher-directed condition performed the best on the posttest, followed by the 25% teacher directed condition.

achievement

Finding Use focused instruction³¹
 along either individual incentives or
 competitive incentives
 rather conventional instruction
 for teaching unspecified mathematics
 content
 at grade seven
 with inner city students

Short Citation Beady & Slavin, 1981

Focus Effects of pretest feedback on
 mathematics (and physical science)
 achievement

Finding There is no advantage to using
 pretest feedback

OR

lesson overviews
 rather than no pretest feedback or no
 overviews
 for teaching unspecified mathematics
 content
 at grade nine
 with regular classroom students

Short Citation Lashier & Wren, 1977

Focus Role of skills analysis in CBM math
 instruction

Finding Use skills analysis in conjunction
 with CBM

³¹ "Focused instruction" is a particular instructional model, similar to the Good and Grouws Missouri Mathematics Project, but more highly structured.

rather than conventional special
education instruction
for basic math facts
at grades three through nine
with students with learning
disabilities

Short Citation Fuchs, Fuchs, Hamlett, & Stecker, 1990

Focus Effects of a collaborative structure-
plus-writing approach to teach
addition and subtraction word problems

Finding Use explicit problem-solving
structure³² instruction and writing
along with teacher-directed
instruction and cooperative learning
groups
rather than problem-solving
instruction alone

OR

rather than conventional instruction
for verbal problem solving
for ordering fractions
at grades three and four
with regular classroom students

Short Citation Rudnitsky, Etheredge, Freeman, & Gilbert,
1995

Focus Effects of teacher + peer modeling on
the achievement of low performing
students

Finding Use successful peer models³³

³² Students were explicitly taught the structure of specific types of problems, such as combine problems and compare problems.

rather than teacher models or no model

OR

Use teacher models
rather than no model
for subtraction with regrouping
at grades two through four
with low regular classroom students

Short Citation Schunk & Hanson, 1985

Studies Related to Computers and Technology

Calculators

Focus Use of calculators in conjunction with teaching computation

Finding Use calculators rather than no calculators for teaching basic whole number computation at (no grade levels given) ages 9-14 with low socioeconomic, ESL, low performing, and average performing students

Short Citation Schnur & Lang, 1976

Focus Effects of using two types of calculators to teach computation

Finding Use "un-programmed" calculators rather than programmed calculators

³³ Students observed video tapes of a student successfully learning an algorithm for subtracting with regrouping. The authors suggest that this was more effective than the teacher model because learners could identify better with the student in the peer model.

OR

Use programmed calculators
rather than no calculators
for mixed computations
at grade three
with regular classroom students

Short Citation Standifer & Maples, 1981

Focus Effects of calculator use on problem-solving performance

Finding Use calculators
rather than no calculators
along with problem-solving instruction
for problems involving decimal
fractions
at grade six
with regular classroom students³⁴

Short Citation Wheatley, 1980

Focus Effects of using calculators and
problem-solving instruction on
problem-solving achievement

Finding Teach problem-solving strategies³⁵
with or without calculators
rather than conventional instruction
for teaching problem-solving
at grade seven
with regular classroom students

Short Citation Szetela, 1987

³⁴ In a university town.

³⁵ The strategies taught were: guessing and testing, making a systematic list, making a simpler problem, searching for a pattern, and drawing a diagram.

Focus Effects of using a calculator on fractions computations

Finding Use calculators to convert fractions to decimals and then perform fractions computations
 along with mastery learning
 rather than conventional fractions computation algorithms
 for fractions computation
 at grade nine
 with regular classroom students

Short Citation Gaslin, 1975

Logo

Focus Effects of using Logo to teach the concept "variable"

Finding Use Logo
 rather than textbooks
 for teaching the concept "variable"
 at grade six
 with metropolitan students

Short Citation Ortiz & MacGregor, 1991

Focus Effects of using Logo to teach motion geometry

Finding Use Logo
 rather than a motion approach
 without Logo

OR

a conventional approach

for teaching geometry
 at fifth and sixth grade
 with regular classroom students

Short Citation Johnson-Gentile, et al., 1994

Focus Effects of Logo versus CAI on problem-solving

Finding Use Logo
 along with teacher-directed initial
 instruction and working in pairs
 rather than CAI
 for problem solving
 at grades four and six
 with regular classroom students

Short Citation Nastasi, Battista, Clements, 1990

Computer-Assisted Instruction (CAI)

Focus Cost and effects of using
 microcomputer instruction for drill
 and practice

Finding Use microcomputers
 along with students working in pairs
 rather than conventional instruction
 for computation
 at grades three and five
 with rural students

Short Citation Fletcher, Hawley, & Piele, 1990

Focus Effects of immersion in CAI on
 mathematics achievement

Finding There is no practical advantage to

using immersion in CAI
rather than conventional instruction
for teaching unspecified content
at sixth grade
with regular classroom students

Short Citation Ferrell, 1986

Focus Effects of learner versus program
control on geometry achievement

Finding Use learner control in CAI programs
rather than program
for geometry
at grades nine and ten
with students ranging from low to high

HOWEVER

Learner control was significantly
higher than program control, but
overall achievement under learner
control was not great

Short Citation Hannafin & Sullivan, 1995

Focus Effects of CAI versus teacher-directed
instruction on teaching number
recognition

Finding Use CAI
rather than teacher-directed
instruction
for teaching numbers and counting
at kindergarten
with students already familiar with
numbers and counting

AND

Use teacher-directed instruction
rather than CAI
for teaching numbers and counting
at kindergarten
with students just beginning to learn
numbers and counting

Short Citation McCollister, Burts, Wright, & Hildreth, 1986

Focus Effects of working in pairs (on a
computer) versus working individually

Finding Use students working on CAI in pairs
rather than working on CAI
individually
for unspecified content
at grade six
with low SES students

Short Citation Mevarech, Silber, & Fine, 1991

Focus Effects of computer programming
instruction³⁶ on mathematics
achievement

Finding Use computer programming
rather than no programming
for unspecified content
at grades seven and eleven
with students at a university high
school

Short Citation Hatfield & Kieren, 1972

Focus Effects on variations of CAI on the

³⁶ The programming was related very directly to specific mathematics concepts and procedures. For instance, students worked on developing a computer program to factor numbers, and were subsequently assessed on their ability to factor numbers (without the aid of the computer).

mathematics achievement of students
with LD

Finding

Use CAI
with or without explicit strategy
instruction or animations
for problem solving
at grades one through six
with students with learning
disabilities

Short Citation

Shiah, Mastropieri, & Scruggs, 1995

Focus

Effects of using computers to teach
fractions

Finding

Use CAI
along with fractions strips
rather than conventional instruction
for teaching fractions
at grade four
with regular classroom students

Short Citation

Ball, 1988

Focus

Effects of using CAI games or drill
and practice on math facts acquisition
for students with learning
disabilities

Finding

Use CAI games

OR

Use drill and practice
rather than no CAI or no drill and
practice
for teaching basic arithmetic facts

at grades four through six
with students with learning
disabilities

Short Citation Okolo, 1992

Focus Effects of using software tools on
problem solving in secondary school

Finding Use computer-assisted guided discovery
along with heuristics and strategy
instruction
rather than overt teacher-directed
heuristics and strategy instruction
for problem solving
at grades nine through twelve
with average students

AND

Use overt teacher-directed heuristics
and strategy instruction
rather than computer assisted guided
discovery
for problem solving
at grades nine through twelve
with lower performers

Short Citation Mayes, 1992

Studies Related to Grouping

Focus Effects of small and large group
instruction on achievement

Finding Train students to work in small groups
along with direct instruction
rather than no training to work in
small groups

for fractions and division
 at fifth grade
 with low and high performing students

IN OTHER WORDS

low and high performing students
 achieved best in small groups when
 they were trained on how to work in
 such groups

Short Citation Swing & Peterson, 1982

Focus Effects of heterogeneous and
 homogeneous cooperative instruction on
 achievement

Finding Use heterogeneous grouping
 along with CAI
 for teaching contrived mathematics
 content
 at grades six and seven
 with low performing students

AND

Use homogeneous grouping
 along with CAI
 for contrived mathematics content
 at grades six and seven
 with high performing students

Short Citation Hooper & Hannafin 1991

Focus Ability times treatment interaction
 effects on children's learning in
 large and small groups

Finding There is no difference between large
 group and small group instruction

along with teacher directed
instruction
for geometry
at fourth and fifth grades
with a broad range of students

HOWEVER

Use small groups
along with teacher-directed
instruction
rather than large groups
for geometry
at fourth and fifth grades
with high and low performers

Short Citation Peterson, Janicki, & Swing, 1981

Focus Effects of whole group versus two-
group teaching

Finding Use whole group instruction
along with active learning³⁷
rather than two groups within a
classroom
for unspecified content
at grades four through six
with an average distribution of
"regrouped"³⁸ learners

Short Citation Mason & Good, 1993

Focus Effects of individualized versus whole
group instruction in bilingual

³⁷ Active learning refers to an instructional approach with several fixed elements, including: prerequisites, perceptions, representation, generality of concepts, relationships, active assessment of student understanding, emphasis upon classroom management, time on task, accountability, reinforcement, peer tutoring, and enrichment.

³⁸ Students were first grouped in whole, relatively homogeneous classrooms

education

Finding There is no difference between individualized and whole group instruction for unspecified mathematics content at grade two with bilingual students

Short Citation Parr, Baca, & Dixon, 1981

Studies Directly Related to Reinforcement Or Motivation Systems

Focus Effects of reinforcement of either academic-facilitative behavior or academic performance (or both) on mathematics

Finding Directly reinforce academic performance, or reinforce behavior consistent with academic performance, or both along with Sullivan Math (1966) rather than no reinforcement for unspecified mathematics content at grades one through three with average and low performers

Short Citation Walker & Hops, 1976

Focus Effects of an experimental resource room with a highly structured extrinsic reward system for attending to academic tasks, for students with behavior disorders

Finding Use a highly structured reinforcement system

along with a pullout setting
 rather than no such system in a
 regular classroom
 for teaching unspecified content
 at grades two through six
 with students who have behavioral or
 emotional disorders

Short Citation Glavin, Quay, Annesley, & Werry, 1971

Focus Effects of goal structures and
 performance contingencies³⁹ on math
 achievement for students with learning
 disabilities

Finding Use self-selected goals
 rather than assigned goals
 for mathematics computation
 at grades nine through twelve
 with students who have learning
 disabilities

Short Citation Fuchs, Bahr, & Rieth, 1989

Focus Effects of teacher versus student goal
 formulation on mathematics achievement

Finding There is no difference between self-
 selected and teacher-selected goals,
 in terms of mathematics achievement
 for unspecified mathematics content
 at grade eight
 with regular classroom students

Short Citation Hannafin, 1981

³⁹ Whether or not a performance contingency—playing a video game—influenced achievement was also studied. There was no significant difference between those students who had to reach goals to play the game, versus those who could play the game non-contingently upon goal acquisition. However, students were not allowed to choose their own reward.

Idiosyncratic Studies

Studies Related to Assessment

Focus Effects of CBM and expert computer-based systems on achievement

Finding Use CBM⁴⁰ and expert computer-based systems along with teacher directed instruction rather than CBM alone

OR

conventional special education instruction for unspecified content at grades two through eight with mildly or moderately handicapped students, or students with learning disabilities or emotional disturbances with regular classroom students

Short Citation Fuchs, Fuchs, Hamlett, & Stecker, 1991

Children's Literature

Focus Effects of using children's literature to improve mathematics achievement

Finding Use children's literature along with child-initiated play and manipulatives rather than conventional instruction

⁴⁰ Curriculum-based Measurement (CBM) is a system entailing frequent assessment of student progress, and frequent adjustments of instruction based upon the assessment. An expert computer-based system provides specific, detailed, "expert" instructional strategies for teachers to use to improve student performance.

for mixed mathematics concepts
 at kindergarten
 with regular classroom and low SES
 students

Short Citation Jennings, Jennings, Richey, & Dixon-Krauss,
 1992

Lattice Algorithm

Focus Multidigit multiplication

Finding Use the "lattice algorithm"
 rather than the conventional multi-
 digit multiplication algorithm
 for multi-digit multiplication
 at grade four
 with regular classroom students

Short Citation Hughes & Burns, 1975

Part-Part-Whole

Focus Using a part-part-whole approach for
 teaching number

Finding Use a part-part-whole
 rather than a standard approach⁴¹
 for teaching numerals
 at kindergarten
 with middle class students

Short Citation Fischer, 1990

⁴¹ The standard method used in the control is identified as a procedure in which students count, say the numeral, and then write it. The part-part-whole approach emphasized the subsets that make up a set, and de-emphasized the writing of numerals.

Homework

Focus Effects of homework on student success in computation with fractions

Finding There is no benefit to assigning homework rather than no homework for fractions computation at grade six with middle class students

Short Citation Gray & Allison, 1971

Matching Amount of Instruction to Learner Preferences for Amount of Instruction

Focus Effects of matching amount of instruction to learner preferences

Finding There is no achievement benefit to matching amount of instruction to learner preferences for amount of instruction rather than specifying amount of instruction for geometry at grades nine and ten with average and lower performing middle-class students

Short Citation Hannafin & Sullivan, 1996

Teaching Complex Fractions

Focus Effects of using associative property versus complex fractions⁴² for teaching division of fractions

⁴² The complex fractions approach requires learning the identity principle, and how to multiply a complex fraction to another complex fraction equal to one.

Finding Use complex fractions rather than the associative property for teaching the division of fractions at grade six with low and average learners

Short Citation Ingersoll, 1971

Teacher Affect

Focus Effects of teacher attitude on mathematics achievement

Finding Use teachers with positive affect⁴³ along with teacher-directed instruction or CAI for teaching unspecified mathematics content at grades seven and eight with low performing and learning disabled students

Short Citation Moore, 1988

Focus Effects of using a game (Mastermind) to teach formal logic

Finding There is no advantage to using Mastermind⁴⁴ rather than not using Mastermind for teaching formal logic at grades six and eight with regular classroom students

⁴³ Positive affect: provide individual help before and after school, use praise, tokens, snacks, and parties, receive satisfaction from seeing student achievement. Negative affect: use sarcasm, ridicule, and criticism in and out of classroom, known to call students stupid, unhappy with remedial assignment.

⁴⁴ A commercially available game.

Variables

For each study, we identified one or more variables and the relative “strength” of each variable. Of course, the variables identified are consistent with the findings outlined above. However, a perusal of the variables gives a somewhat broader view of our findings.

We coded the “strength” of each variable as follows:

SF: Strong support, focus of study. This means that there was strong support for the variable in a study, and that the variable in question was also the focus of the study.

SI: Strong support, implied in study. This also indicates strong support, but the variable in question was not the focus of the study. For a variable to receive this rating, it must have been present in one or more experimental conditions, but not in the control, or present in one or more experimental conditions, but not in other experimental conditions.

SFC and SIC: Same as above, except that the variable was clearly one of two or more variables studied. The relative strength of a clustered variable, then, is less than that of an isolated variable, given that the interactions among clustered variables are not known.

NE: No effect.

PAC: Present in all conditions. A given variable is sometimes identified as being held constant across all instructional conditions. The relative strength of such variables is weak, but can't be discounted entirely, given that the effectiveness of a strong variable could depend in part upon an interdependency with one or more weak variables.

Following is a list of all the variables identified, in the order they were identified, along with the codings of relative strength.

List of Variables

Variable	SF	SI	SFC	SIC	NE	PAC
student team learning	2		1			
group external rewards		1	1	1	1	
positive teacher attitude	1					
external rewards	2	1	2	5	1	1
scaffolding	1			7		
active participation	1		2	4		1
feedback with corrections			1	10		
enhanced cues			1			
lattice algorithm	1					
CAI fractions strips	1					
computer programming	1					
computer-assisted instruction for non-initial instruction	1	1				
computer assisted computation	2				1	
computer tools for low performers					1	
computer tools for average performers			1			
computer tools for high performers					1	
teacher-directed instruction 25% or 75% of class time	1					
focused instruction			1	2		
clear specification of objectives				1		
explicit teacher-directed initial instruction	1			6		8
explicit initial instruction - teacher or computer						3
fewer tasks - more depth			1			
cooperative homework teams					1	
homogeneous cooperative grouping for high performers	1					

Variable	SF	SI	SFC	SIC	NE	PAC
heterogeneous cooperative grouping for low performers	1					
student-centered discourse			1			
train students for small group instruction for high and low performers	1		1			
small group instruction				2		
whole group instruction	1			1	1	
whole group for medium ability students			1			
individualized instruction	1		1		1	
peer-assisted learning			1			
students working in pairs		1				1
peer modeling			1			
peer tutoring	1		3	2		
high structure reciprocal peer tutoring			2	3		1
parental involvement			2			
number families for facts				2		
counting and thinking strategies for facts	2					
counting on strategy for facts					1	
rote strategy for facts					1	
mastery-based cooperative learning	1					
mastery learning	2				1	
mastery before moving on				5	1	
pull-out setting				1		
overt strategy instruction			1	4		
number families			1			
sequence easy-to-difficult			1			1
part-part-whole for numbers	1					
expository instruction				1	1	
discovery instruction					1	
didactic instruction for higher performers				1		

Variable	SF	SI	SFC	SIC	NE	PAC
didactic instruction for lower performers	1		1	1		
guided discovery				1		
guided discovery for higher performers	1			2		
didactic instruction for learning concepts	1					
guided discovery for learning principles	1					
no homework					1	
homework					1	
match amount of instruction to learner preferences					1	
learner control	1					
program control		1		1		
cooperative learning	2		1	1	2	
competitive learning					2	
individualistic learning	1				1	
deductive for limit in calculus					1	
deductive for derivative in calculus	1					
inductive for limit in calculus					1	
self-instructional training	1		1			
pictorial representations					1	
manipulatives	1				2	
teacher modeling			1			
explicit teacher-directed instruction				6		
use manipulatives						1
use manipulatives with computation					1	
use word problems with computation					1	
use of Logo to teach problem-solving	1					
use of Logo to teach concept "variable"	1					

Variable	SF	SI	SFC	SIC	NE	PAC
use small group games to teach number concepts	1					
use calculators to teach fractions computations	1					
use calculators to teach problem-solving	1				1	
use calculators to teach computation	2					
integrative and elaborative instruction	1					
rational sets to teach concepts	1					
teacher models strategy				6		
feedback on mental modeling			1	1		
frequent assessment				4		
curriculum-based measurement			3			
expert system - computer-based			1			
complex fractions for division	1					
heuristics for problem-solving	1					
children's literature	1					
practice to learn facts	1					
strategies to learn facts				1	1	
strategies to learn problem-solving				2	1	
animations in CAI to learn problem-solving					1	
games to teach formal logic					1	
process problem-solving strategy					1	
pretest feedback					1	
transformational geometry instruction	1					
overt metacognitive problem-solving instruction	1			1		
highly structured instruction				2		1
synthetic or combined synthetic and analytic	1					

Variable	SF	SI	SFC	SIC	NE	PAC
self-selected goals	1					
skills analysis			1			
write to learn for problem-solving			1			
problem categories, e.g. change problems				1		
logical sequence	1					
teacher selects goals					1	
student selects goals					1	
active mathematics teaching						1
rational sets of examples	1					
use of Logo to teach motion geometry	1					
games	1					
computer-based remediation			1			
fluency building (speed drills)				1		
computer-assisted problem-solving	1					
programmed learning	1					
pre-instructional overview					1	
drill and practice	1					
guided discovery for average performers			1			
monitoring of student progress	1					
self-monitoring of student progress					1	
adult monitoring of student progress					1	

List of Grouped Variables

External Rewards.

Variable	SF	SI	SFC	SIC	NE	PAC
external rewards	2	1	2	5	1	1
group external rewards		1	1	1	1	
positive teacher attitude	1					
student team learning	2		1			

Collaborative Peer Work.

Variable	SF	SI	SFC	SIC	NE	PAC
cooperative homework teams					1	
homogeneous cooperative grouping for high performers	1					
heterogeneous cooperative grouping for low performers	1					
peer-assisted learning			1			
students working in pairs		1				1
peer modeling			1			
peer tutoring	1		3	2		
high structure reciprocal peer tutoring			2	3		1
cooperative learning	2		1	1	2	
competitive learning					2	
individualistic learning	1				1	

Didactic (expository, explicit), guided discovery, and discovery (implicit) instruction

Variable	SF	SI	SFC	SIC	NE	PAC
deductive for derivative in calculus	1					
deductive for limit in calculus					1	
inductive for limit in calculus					1	
didactic instruction for higher performers				1		
didactic instruction for lower performers	1		1	1		
didactic instruction for learning concepts	1					
discovery instruction					1	
enhanced cues			1			
explicit teacher-directed instruction				6		
explicit teacher-directed initial instruction	1			6		8
explicit initial instruction - teacher or computer						3
expository instruction				1	1	
focused instruction			1	2		
guided discovery				1		
guided discovery for average performers			1			
guided discovery for higher performers	1			2		
guided discovery for learning principles	1					
overt metacognitive problem-solving instruction	1			1		
overt strategy instruction			1	4		
synthetic or combined synthetic and analytic	1					
teacher modeling			1			
teacher models strategy				6		

Computers and Calculators

Variable	SF	SI	SFC	SIC	NE	PAC
animations in CAI to learn problem-solving					1	
CAI fractions strips	1					
computer assisted computation	2				1	
computer-assisted instruction for non-initial instruction	1	1				
computer-assisted problem-solving	1					
computer tools for low performers					1	
computer tools for average performers			1			
computer tools for high performers					1	
computer programming	1					
computer-based remediation			1			
use calculators to teach computation	2					
use calculators to teach fractions computations	1					
use calculators to teach problem-solving	1				1	
use of Logo to teach concept "variable"	1					
use of Logo to teach motion geometry	1					
use of Logo to teach problem-solving	1					

Grouping

Variable	SF	SI	SFC	SIC	NE	PAC
train students for small group instruction for high and low performers	1		1			
small group instruction				2		
whole group instruction	1			1	1	
whole group for medium ability students			1			
individualized instruction	1		1		1	
pull-out setting				1		
use small group games to teach number concepts	1					

Instructional Tasks

Variable	SF	SI	SFC	SIC	NE	PAC
fewer tasks - more depth			1			
rational sets of examples	1					
rational sets to teach concepts	1					
logical sequence	1					
sequence easy-to-difficult			1			1
integrative and elaborative instruction	1					

Instructional Models

Variable	SF	SI	SFC	SIC	NE	PAC
mastery learning	2				1	
mastery before moving on				5	1	
mastery-based cooperative learning	1					
programmed learning	1					
curriculum-based measurement			3			
active mathematics teaching						1

Components of Instructional Approaches

Variable	SF	SI	SFC	SIC	NE	PAC
active participation	1		2	4		1
monitoring of student progress	1					
adult monitoring of student progress					1	
self-monitoring of student progress					1	
clear specification of objectives				1		
drill and practice	1					
expert system - computer-based			1			
feedback on mental modeling			1	1		
feedback with corrections			1	10		
fluency building (speed drills)				1		
frequent assessment				4		
highly structured instruction				2		1
homework					1	
no homework					1	
pre-instructional overview					1	
pretest feedback					1	
scaffolding	1			7		
self-instructional training	1		1			
self-selected goals	1					
skills analysis			1			
student selects goals					1	
student-centered discourse			1			
teacher selects goals					1	
teacher-directed instruction 25% or 75% of class time	1					
write to learn for problem-solving			1			

Other: Math Facts

Variable	SF	SI	SFC	SIC	NE	PAC
number families			1			
number families for facts				2		
part-part-whole for numbers	1					
practice to learn facts	1					
strategies to learn facts				1	1	
counting and thinking strategies for facts	2					
counting on strategy for facts					1	
rote strategy for facts					1	

Other: Problem Solving

Variable	SF	SI	SFC	SIC	NE	PAC
heuristics for problem-solving	1					
use word problems with computation					1	
strategies to learn problem-solving				2	1	
process problem-solving strategy					1	
problem categories, e.g. change problems				1		

Other: Manipulatives

Variable	SF	SI	SFC	SIC	NE	PAC
manipulatives	1				2	
use manipulatives						1
use manipulatives with computation					1	
pictorial representations					1	

Other

Variable	SF	SI	SFC	SIC	NE	PAC
lattice algorithm	1					
parental involvement			2			
complex fractions for division	1					
children's literature	1					
games to teach formal logic					1	
transformational geometry instruction	1					
games	1					

Addendum

Our original report of January 31, 1998 was based upon a review of seventy-seven high quality experimental studies of mathematics. For this addendum, we have reviewed an additional thirty-three articles. Our findings from those additional thirty-three articles are listed below.

With one notable exception, the addition of thirty-three high quality studies has not resulted in significant changes to our findings from the initial seventy-seven articles. The exception involves Curriculum-based Measurement (CBM), a particular assessment system associated most prominently with Fuchs & Fuchs, Deno, and Shinn. In our original report, only one article reported on a CBM study. We categorized that study as “idiosyncratic.” However, three articles from the additional thirty-three also report positive effects for CBM. In addition, one study from the thirty-three found student self-assessment to be more effective than no assessment. Thus, we have added a new category—assessment—to our findings, with five articles in that category.

Otherwise, studies from the additional thirty-three either lent some strength to earlier findings or added some equivocation, or dealt with new but idiosyncratic topics. For example, one of the studies studied the use of calculators for computation. That study was a virtual replication of one of the studies in our original report, except that it was conducted with a different population of students. The findings of both were the same: calculator use had positive effect as a supplement to computation instruction.

As an example of a study that added some equivocation to the findings we reported originally, one study found no effects for CAI when teaching algebra. One new idiosyncratic study investigated the effects of using personalized verbal problems for teaching verbal problem solving.

We have compiled brief descriptions of all one hundred nine studies in a document entitled:

<p>REPORT TO THE CALIFORNIA STATE BOARD OF EDUCATION</p> <p>Review of High Quality Experimental Mathematics Research</p> <p>Executive Summary</p> <p>“At a Glance”</p>
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The findings of the most recent thirty-two studies are as follows:

Studies related to work among peers

Focus	Effects of older peer participant models on deficient academic performance.
Finding	Use a participant peer modeling ⁴⁵ system along with positive reinforcement rather than conventional instruction for unspecified mathematics content at grade eight with low performing students
Short Citation	Horan, De Girolomo, Hill, & Shute, 1974

⁴⁵ The peer modeling system used in this study followed a three-phase structure. Tutors modeled or demonstrated for fifteen minutes, then helped student work through problems for fifteen minutes, and finally had students work through problems alone for fifteen minutes. Tutors corrected any errors made during the final fifteen minutes of independent application.

Idiosyncratic Studies

Focus	Effects of vocabulary instruction in ratio and proportion
Finding	Use vocabulary instruction rather than no vocabulary instruction for ratio and proportion instruction at grade seven with students ranging from high to low socioeconomic status
Short Citation	Jackson & Phillips, 1983
<hr/>	
Focus	Effects of tutoring versus classroom instruction on exponential notation ability of low performing students
Finding	Use tutoring rather than conventional classroom instruction for exponential notation at grades four and five with students ranging from low to high
Short Citation	Bausell, Moody, & Walzl, 1972
<hr/>	
Focus	Effects of mental computation on problem solving and computation
Finding	Use mental computation rather than conventional classroom instruction for problem solving and computation at grade four with regular classroom students
Short Citation	Thompson, Strackbein, & Williams, 1992
<hr/>	
Focus	Effects of personalized word problems on division of fractions word problem-solving

Finding Use personalized word problems⁴⁶
rather than conventional word problems
for solving division of fractions word
problems
at grades five and six
with regular classroom students

Short Citation Ross, Anand, Morrison, & O'Dell, 1988

Strategies

Focus Effects of two instructional
strategies for teaching the solution
to verbal problems

Finding There is no significant difference
between the Polya⁴⁷ and DPPC methods
for verbal problem solving
at grade nine
with private school girls

Short Citation Bassler, Beers, & Richardson, 1975

Focus Effects of a data table strategy for
constructing multiplication and
division sentences from word problems

Finding Use data tables⁴⁸
rather than substitution strategy
for constructing multiplication and
division sentences from proportional
word problems
at grade seven
with regular classroom students

⁴⁶ Personalized word problems were generated by having each student in the experimental condition fill out a biographical questionnaire. A computer then generated the word problems by inserting the individualized contexts (names, birthdates, etc.) into generalized word problems.

⁴⁷ The Polya Method encourages students to look at a problem as a whole, whereas the DPPC method encourages students to translate information as it appears in problems.

⁴⁸ Students were taught to write proportional word problems out in a data table, or "linking table," that showed the proportional relationship between the values.

Short Citation Sellke, Behr, & Voelker, 1991

Focus Effects of using flow charts

Finding There is no difference between the flow chart method and conventional classroom instruction for computation, concepts, and applications in word problem solving at grade nine with low achievers

Short Citation Thompson, 1980

Computers and Technology

Focus Effects of individualized and cooperative computer-assisted instruction

Finding Use either form of CAI for unspecified mathematics content at grade three with **higher** achieving low SES students

AND

Use cooperative CAI for unspecified mathematics content at grade three with **lower** achieving low SES students

Short Citation Mevarech, 1993

Focus Effects of CAI game and CAI "non-game" programs on basic math facts

Finding Use "plain vanilla" CAI⁴⁹

⁴⁹ The findings were strong and unambiguous for students identified with learning disabilities, and only slightly less so with lower performing students.

rather than CAI games
 for basic addition fact fluency
 at (grades not specified)
 with students identified as low-
 performing or learning disabled

Short Citation Christensen & Gerber, 1990

Focus Effects of CAI and traditional mastery
 learning

Finding Use mastery learning
 rather than conventional instruction

AND

Provide remedial instruction in the
 opposite mode (teacher directed or
 CAI) as used in initial instruction
 for simple algebraic computation
 for ordering fractions
 at grade eight
 with regular classroom students

Short Citation Dalton & Hannafin, 1998

Focus Effects of hand-held calculators and
 program-feedback calculators on
 mathematics concepts and computation

Finding Use hand-held calculators
 rather than program-feedback
 calculators

OR

rather than no calculators
 for concepts and computation
 at grade four
 with Title I students
 with regular classroom students

Short Citation Standifer & Maples, 1982

Focus Effects of LOGO and CAI on mathematics achievement

Finding Use LOGO or CAI rather than conventional instruction for unspecified content at grade one with regular classroom students

Short Citation Emihovich & Miller, 1988

Focus Effects of CAI on algebra achievement

Finding There is no difference between CAI and conventional classroom instruction for algebra achievement at grades 10-12 with regular classroom students

Short Citation Saunders & Bell, 1980

Focus Effects of instruction by computer and by video program, together and separately

Finding Use CAI

OR

Use a video program rather than both together, or neither for geometry at grade five with regular classroom students

Short Citation Mevarech, Shir, & Movshovitz-Hadar, 1992

Instructional Design

Focus Aptitude treatment interaction effects of variations of direct instruction

Finding Use whole group direct instruction⁵⁰
with a small group variation
 for fractions instruction
 at grades 4 and 5
 with **internally** motivated students
 who are **higher** performers and who
 have a **positive attitude** toward
 mathematics

AND

Use whole group direct instruction
without a small group variation
 for fractions instruction
 at grades four and five
 with **externally** motivated students
 who are **lower** performers and who have
 a **negative** attitude toward
 mathematics

Short Citation Janicki & Peterson, 1981

Focus Effects of programmed and random
 instructional sequences

Finding There is no difference between
 programmed and random instructional
 sequences
 for fractions instruction
 at ages 15-20
 with students who have been identified
 as educable mentally retarded

Short Citation Cartwright, 1971

Focus Concept learning

Finding Use rational sets of best examples,

⁵⁰ The "direct instruction" used in this study was based upon a model that has been researched by Rosenshine and his associates. The small group variation involved heterogeneous groups of four students working together during time that was allocated for individual work in the direct instruction group without the small group variation. An interesting finding that was not a focus of this study was that the more training teachers had in this model of direct instruction, the better their students performed.

along with definitions
rather than a definition and a
statement of relationships

AND

Use expository and interrogatory
examples
rather than interrogatory examples
alone
for the concept of regular polygon
at grade three
with suburban regular classroom
students

Short Citation Tennyson, Youngers, & Suebsonthi, 1983

Focus Effects of incidental versus
intentional instruction on number
concepts

Finding Use game formats
along with external rewards
rather than conventional instruction
for basic number concepts
at mean age of 92.80 months
with educable mentally retarded
students

Short Citation Ross, 1970

Focus Effects of three methods of
introducing two-digit numeration

Finding Use grouping objects in sets of ten
and counting by tens
rather than other approaches
for two-digit numeration
at kindergarten
with rural students

Short Citation Barr, 1978

Focus	Effects of teacher's written feedback on student homework
Finding	Use specific teacher's comments (feedback) on students' homework rather than number of answers correct for reasoning or applications with fractions, numbers, and geometry at grade six with regular classroom students
Short Citation	Cardelle-Elawar & Corno, 1985
Focus	Effects of the Piacceleration program
Finding	Use the Piacceleration instructional method ⁵¹ along with a wide range of exemplars and the criterion of full mastery rather than conventional instruction for classification, seriation, and number conservation at kindergarten with lower performing children
Short Citation	Pasnak, Holt, Campbell, & McCutcheon, 1991
Focus	Effects of learning goals along with self-evaluation
Finding	Use learning goals along with student self-evaluation of performance capabilities or progress in skill acquisition rather than performance goals for addition and subtraction of fractions at grade four with middle class, average students

⁵¹The Piacceleration method was "developed out of efforts to use learning sets to teach classification, seriation, and number conservation to blind and mild mentally retarded students" (Pasnak, Hold, Campbell, & McCutcheon, 1991). The method is based on Piagetian child development theory.

Short Citation Schunk, 1996

Reinforcement & Motivation

Focus Individual and combined effects of self-monitoring and goal-setting procedures

Finding Use self-monitoring rather than self-monitoring combined with goal-setting

OR

goal-setting alone
for unspecified content
at grades five and six
with regular classroom students

Short Citation Sagotsky, Patterson, & Lepper, 1978

Focus Effects of student self-assessment on mathematics achievement

Finding Use student self-assessment rather than conventional assessment for unspecified mathematics content at grades three and four with regular classroom students

Short Citation Fontana & Fernandez, 1994

Focus Effects of student's self monitoring on achievement

Finding Use self-monitoring or conventional adult monitoring rather than no monitoring of progress for subtraction at ages 8 years 6 months to 9 years 5

months

with regular classroom students

Short Citation Schunk, 1983a (Journal of Experimental Education)

Focus Effects of reward contingencies

Finding Use reward contingencies rather than no reward contingencies for division ages 8 years 8 months to 11 years 6 months with middle class regular classroom students

Short Citation Schunk, 1983b (Journal of Educational Psychology)

Grouping

Focus Effects of group size on mathematics performance

Finding Use small classes (15) rather than larger classes (22) with a teacher and aide for unspecified mathematics content at kindergarten with inner-city students

Short Citation Finn, Achilles, et al, 1990

Focus Effects of three grouping methods

Finding Use team-assisted individualization

OR

Use within-class ability grouping

rather than a whole class approach
 for computation, concepts, and
 applications
 at grades three through six
 with regular classroom students

Short Citation Slavin & Karweit, 1985

Focus Effects of large- and small-group
 teaching approaches

Finding Use small group cooperative
 instruction
 rather than large group individualized
 instruction
 for fractions
 at grades four through six
 with higher performing students

AND

Use large group individualized
 instruction
 rather than small group cooperative
 instruction for fractions at
 grades four through six with
 higher performing students
Short Citation Peterson & Janicki,
 1979

Assessment

Focus Effects of extended CBM
 decision-making strategies on
 students' mathematics
 operations **Finding** Use
 CBM along with instructional
 strategies based on CBM
 results rather than CBM without
 strategies
 for fractions at grades four
 through six with higher
 performing students
Short Citation Peterson & Janicki,
 1979

Assessment

Focus Effects of extended CBM
 decision-making strategies on

Assessment

Focus Effects of extended CBM decision-making strategies on students' mathematics operations
Finding Use CBM along with instructional strategies based on CBM results rather than CBM without strategies

OR

rather than no CBM for mathematics operations at grades two through five with LD, low, and regular classroom students

Short Citation Fuchs, Fuchs, Hamlett, Phillips, & Bentz, 1994

Focus Effects of goal line feedback

Finding Use goal line feedback along with CBM tests and accurate goal feedback interpretation training line rather than no goal line feedback for computation at grades two through eight with students identified with LD

Short Citation Fuchs, Fuchs, Hamlett, & Whinnery, 1991

Focus Effects of reintegration methods

Finding Use CBM rather than traditional and transenvironmental programming⁵²

⁵² Transenvironmental reintegration includes four phases: 'environmental assessment' (to ascertain skills and behaviors required for success in regular classroom), 'intervention and preparation' (to teach the skills identified in previous phase), 'promoting transfer across settings' (to help ensure the students uses the skills in regular education), and 'evaluation in the mainstream' (to collect data on the extent to which the student has adjusted academically and socially) (Fucks, Roberts, Fuchs, & Bowers, 1996).

OR

rather than transenvironmental
programming only
for unspecified mathematics content
at grades three through seven
with students identified with LD

Short Citation

Fuchs, Roberts, Fuchs, & Bowers, 1996
